Writing Class 2: The Summary: The Most Important Page

Mathematical Modeling Lecture

Mathematical Modeling

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Session Route

Launch	Build	Test
Question Motivating context, key vocabulary, and the first modelling decision.	**Method** Worked example, diagram or table, calculation, and parameter meaning.	**Critique** Checkpoint practice, limitation check, and communication of results.

Columns are used where comparison helps; explanation slides stay clean and spacious.

Why the Summary Decides Everything

Core explanation, worked reasoning, and application

Why the Summary Decides Everything

In W1 we noted that judges spend disproportionate time on the first page. Now we say it more directly: the Summary often determines whether your paper is read carefully at all.
Consider the judging process for a major competition. A single judge may evaluate 30–50 papers in a few days. They cannot read every page of every paper with equal attention. What they do is the following:

- Read the Summary carefully (5–10 minutes).
- Based on the Summary, form a tentative ranking.
- Skim the rest of the paper to confirm or revise.
- Final ranking is heavily anchored by Summary impressions.

This is not laziness. It is the only feasible procedure given time constraints. The Summary serves as both an **advertisement** and a **filter**: it advertises what your paper accomplishes and filters whether the rest deserves close attention.

Note: The Disproportionate-Effort Rule

In a 36-page paper, the Summary is approximately 2.7% of the length but plausibly determines 40–60% of the final outcome. Spend a corresponding share of your writing time on it. Many teams spend 30 minutes on the Summary at the end of the 96-hour competition. **This is the single most leverage-rich mistake to fix.**

The Four-Block Structure

Core explanation, worked reasoning, and application

The Four-Block Structure

Although every Summary is different, almost every *good* Summary follows a four-block structure. Internalising this structure is the single most useful Summary-writing skill.

Method: Method: The Four-Block Summary Structure

1. **Problem block** (2–4 sentences): What is the problem? Why does it matter? What is asked of you specifically?
2. **Approach block** (3–5 sentences): What is your approach? Which models did you build, in what order, why? Mention 2–3 specific techniques.
3. **Results block** (3–5 sentences): What did you find? Give *numbers*. Make the headline answer impossible to miss.
4. **Insight / Recommendation block** (2–3 sentences): What does this mean? What is your recommendation? Why should the reader believe you?

Total: 10–17 sentences, fitting on one page with single spacing or comfortable double spacing.

Note: A Useful Diagnostic

After drafting your Summary, label every sentence with one of P (Problem), A (Approach), R (Result), I (Insight). The labels should appear roughly in this order. If your Summary is mostly A's with no R's, you have described *what you did* but not *what you found* — a fatal flaw.

Block 1: Problem

This block proves you understood the question. Three moves to make:

- Briefly establish **context** (one sentence).
- State the **specific question** asked (one sentence; in your words, not theirs).
- Mention what makes the problem **interesting or difficult** (one sentence).

Worked Example: Example 1: Problem Block — HiMCM-Style Problem on Drone Light Shows

Weak:

We were asked to figure out how to make a drone light show. The problem is from HiMCM 2024.

Better:

Drone light shows are increasingly replacing fireworks for large public celebrations because they produce no air or sound pollution and allow programmable imagery. The central modeling challenge is to determine how many drones are needed to render a target image at a given altitude and viewing angle, and how to assign each drone an efficient flight path that avoids collisions. This combination of geometric resolution and combinatorial routing is what makes the problem non-trivial.

Note: What Made the Second Version Stronger

- Opens with context (drones replace fireworks) before diving in.
- Restates the question in modeling language (‘geometric resolution and combinatorial routing’).
- Names what makes it hard, signalling that the team thought about the problem rather than just receiving it.
- Does not mention the contest year — judges know what year it is.

Block 2: Approach

This block proves you built something. Three moves to make:

- State your **strategy** at one level of abstraction higher than the equations (one sentence).
- Briefly describe **each model** you built and how they relate (two or three sentences).
- Indicate **which techniques** were used (one or two sentences naming concrete methods).

Worked Example: Example 2: Approach Block

Weak:

We used MATLAB and Python to make a model. We tried different things and picked the best one. Our model uses lots of equations and computer simulations to find the answer.

Better:

We approach the problem in three stages of increasing realism. Model 1 computes the minimum number of drones needed to render a target image at a fixed viewing distance, using a discrete image-sampling argument. Model 2 extends this to allow drones to move during the display, transforming the question into a min-cost assignment problem solvable by the Hungarian algorithm. Model 3 adds collision-avoidance via a velocity-obstacle method validated by Monte Carlo simulation of 10^4 *random initial conditions.*

Note: Why the Second Version Works

- Numbered models give a clear progression and let the reader hold them in memory.
- Specific algorithms (Hungarian, velocity-obstacle, Monte Carlo) signal mathematical depth without losing the reader.
- The number 10^4 commits to a concrete simulation effort.
- Every claim is something the body of the paper must defend; do not promise more than you delivered.

Block 3: Results

This block delivers the **headline answer**. The single most important sentence of your entire paper lives here. It must:

- Be a **quantitative claim with units**.
- Be the **first thing** stated in the block, not buried after methodology recap.
- Be **visually emphasised** where appropriate (bold, italics, or a result table).

Worked Example: Example 3: Results Block — the Headline Sentence

Weak (headline buried):

> *After running our simulations and analysing the results, we found values for various parameters and arrived at a recommendation regarding the optimal configuration.*

Better (headline first):

> *We recommend deploying 312 drones, with assigned paths totalling 2,840 metres and a maximum collision rate of 0.02% across 10^4 simulated runs. Our two upper bounds (a geometric ceiling of 350 drones and an information-theoretic ceiling of 295) bracket this recommendation and indicate it is within 6% of the theoretical optimum.*

Warning: Warning: Hide Nothing in the Results

A judge should be able to find your answer in fewer than 10 seconds by glancing at the Summary. If they have to read three paragraphs and still are not sure what the answer is, the Summary has failed — regardless of how clever the model is.

Block 4: Insight / Recommendation

This block elevates the work from a numerical exercise to a defensible recommendation. Two moves:

- State **why** the recommendation is trustworthy (cite validation or sensitivity briefly).
- State the **broader implication** or recommendation in plain language.

Worked Example: Example 4: Insight Block

Weak:

In conclusion, we believe our model gives a good answer to the problem. Future work could improve the model.

Better:

Sensitivity analysis shows the recommended fleet size is robust to $\pm 15\%$ variation in audience-distance assumptions, and that the dominant constraint at scale is communication-latency, not propulsion. Event organisers planning shows of similar scope should therefore allocate budget toward redundant communication channels rather than additional drones.

A Complete Worked Summary

Core explanation, worked reasoning, and application

A Complete Worked Summary

Below is a full Summary against the (imagined) problem: *Model the effect of installing a network of bicycle highways on commute times in a mid-sized city.*

Worked Example: Example 5: Complete Summary — Bicycle Highway Network

PROBLEM. As mid-sized cities seek to reduce traffic congestion and emissions, several have begun investing in dedicated bicycle highways — continuous, signal-free, motor-traffic-free bike paths between major centres. We are asked to determine how such a network should be designed for a city of 200,000 residents, and to estimate the resulting reduction in average commute time. The problem combines network design, mode-choice modeling, and uncertainty in commuter behaviour.

APPROACH. We attack the problem in three stages. **Model 1** treats the city as a graph and solves a minimum-spanning-network problem to identify the most cost-effective set of highway routes connecting the eight largest residential clusters to the central business district. **Model 2** introduces a **discrete-choice mode model** (logit) calibrated against transport-survey data, predicting what fraction of commuters will switch from cars to bicycles given a new highway. **Model 3** feeds the predicted switch rates back into a **traffic flow model**, allowing us to estimate residual car congestion and overall commute time savings. We validate against published European deployments (Copenhagen, Utrecht).

RESULTS. Our recommended network consists of **four bicycle highways totalling 47 km**, built at an estimated cost of **EUR 28 million**. This network is predicted to shift **18%** of car-based commuters to bicycles, reducing peak-hour car traffic by **14%** and the average commute time by **6.2 minutes per traveller**. Sensitivity

Worked Example: Example 5: Complete Summary — Bicycle Highway Network

analysis indicates the modal shift estimate is most sensitive to weather assumptions (range 14–22%) and least sensitive to construction cost; the savings recommendation is therefore robust.

INSIGHT. The break-even period of the investment is approximately **4.3 years** on commute-time savings alone, before considering health and emissions benefits. City planners should prioritise highways connecting the two highest-density residential clusters to the city centre first, as these account for 62% of total time savings in our model. Our results are consistent with the European data and suggest that bicycle highways are an unusually high-return urban transportation investment for mid-sized cities.

Note: How to Read This Summary

- **Problem block:** 3 sentences. Context (mid-sized cities reducing emissions), question (network design + commute reduction), difficulty (network + mode choice + behaviour).
- **Approach block:** 5 sentences. Three numbered models, two specific techniques named (logit, traffic flow), validation source mentioned.
- **Results block:** 4 sentences. *Five* bolded quantitative claims. Sensitivity touched.
- **Insight block:** 3 sentences. Break-even period, prioritisation, broader implication.

Total: 15 sentences. Fits on one page.

Diagnosing Common Summary Failures

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Diagnosing Common Summary Failures

We now look at the four most common failure patterns, each with a real-style example and a fix.

Worked Example: Failure 1: The Methods-Heavy Summary (most common)

Failure:

> *We used a system of differential equations together with the Runge-Kutta method to solve them. We also performed a Monte Carlo simulation with 10,000 trials. The results were then analysed using regression and the model was validated.*

Diagnosis: The reader learns *nothing* about the problem or the answer. This is an Approach block with no Problem, Results, or Insight.

Fix: Move this content into Block 2 (Approach), shrink it to one or two sentences, and add Blocks 1, 3, and 4.

Worked Example: Failure 2: The Process Narrative

Failure:

> *Our team first read the problem and discussed possible approaches. We then divided the work and each member tackled a different aspect. After several iterations, we converged on a final model, which we then refined to produce our recommendations.*

Diagnosis: The reader learns about the team's *workflow*, not about the modeling. This is almost always the result of writing the Summary last and confusing it with a reflection or acknowledgement.

Fix: Delete the entire paragraph. Replace with content about the problem, model, and findings.

Worked Example: Failure 3: The Buried Answer

Failure:

> *This paper investigates several models for the optimal coffee-shop staffing problem. After developing three models, performing sensitivity analysis on key parameters, validating our results against the literature, and considering implementation constraints, we have produced a final recommendation that aligns with the expectations of the operational research literature.*

Diagnosis: 50 words, zero numbers. No answer.
Fix: The very next sentence must be: *The optimal staffing level is 3 baristas during peak hours and 1 during off-peak.*

Worked Example: Failure 4: The Hyped Summary

Failure:

> *Our novel and innovative model uses state-of-the-art techniques including deep learning, fuzzy systems, and genetic algorithms to perfectly solve the problem. Our results are highly accurate and our recommendations are guaranteed to optimise the situation.*

Diagnosis: Adjectives doing the work that numbers should do. Triggers immediate skepticism in judges, who have seen exactly this kind of language fail many times.

Fix: Replace every adjective with a number. "novel" $\rightarrow$ describe what makes the approach different in one sentence. "state-of-the-art" $\rightarrow$ drop. "perfectly solve" $\rightarrow$ "within 5% of the theoretical optimum". "highly accurate" $\rightarrow$ "validated against 12 data points with mean error 4%".

Writing Process: Drafting the Summary

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Method: Method: When and How to Write the Summary

1. **First draft: end of Day 1** (or as soon as you have a working baseline model). This forces you to articulate the problem and approach before the details overwhelm.
2. **Second draft: end of Day 2/3** after results are available. Replace placeholder numbers with actual numbers.
3. **Third draft: in the last** 4–6 **hours.** Polish for language. Read aloud. Ask: would a stranger find the answer in 10 seconds?
4. **Final test:** have a teammate who did not write the Summary read it. Ask them: (a) what is the question? (b) what is the answer? (c) why should we trust it? If they cannot answer all three, revise.

Warning: Warning: Common Process Mistakes

- **Writing the Summary only at the very end.** The Summary written at hour 94 of 96 is almost always weak.
- **Letting one person write the whole Summary alone.** The Summary is too important; at minimum, two teammates should review.
- **Copying sentences from the body.** The Summary is its own document. Its sentences are tighter, more declarative, more headline-like than body text.

Method: Method: Write Three Versions, Pick the Best Pieces

For a high-stakes Summary, write three full drafts:

1. **Version A: methods-first.** Lead with the Approach. *Strength:* shows mathematical depth early.
2. **Version B: results-first.** Lead with the Result. *Strength:* maximum impact; judges immediately know your answer.
3. **Version C: problem-first.** Lead with the Problem (classic structure). *Strength:* demonstrates understanding; smooth read.

Read all three. Pick the best Problem block, the best Approach block, the best Results block, and the best Insight block, then assemble. The final draft is usually closest to C, with the Results block from B.

Workshop Exercises

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Exercises: Exercise W2.1: Label the Sentences

Take any of the example Summaries in this class. For every sentence, label P (Problem), A (Approach), R (Result), or I (Insight). Then count the sentences in each block. Compare against the recommended 2–4, 3–5, 3–5, 2–3.

Exercises: Exercise W2.2: Diagnose and Rewrite

The following is a (constructed) weak Summary. Diagnose every problem and rewrite it as a strong four-block Summary.

> *Our team studied the problem of optimal bus routing for our school district. We developed a model using mathematical techniques. After many iterations, we determined that our model works well and produces results that are useful for the school district. We hope our recommendations will be considered. Future work could extend our approach.*

Assume (for the rewrite) that the actual findings were: 4 buses needed, total daily route length 87 km, average student commute reduced from 43 to 28 minutes, sensitivity test shows the recommendation is stable to ± 10 students per route.

Exercises: Exercise W2.3: Write Three Versions

Choose any modeling problem from your past coursework (or a HiMCM problem you have read). Write three Summary drafts of approximately 15 sentences each, following Method 5.1 (methods-first, results-first, problem-first). Have a teammate read all three blindly and rank them.

Exercises: Exercise W2.4: The 10-Second Test

Find one Outstanding paper online. Open it to the Summary page. Time yourself: how long does it take you to identify the team's headline numerical answer? Now do the same with your most recent Summary. If yours is slower, redesign.

Exercises: Exercise W2.5: The Length-Cut Drill

Take your most recent Summary draft. Without losing any content, cut it by 30%. Methods to try:

- Replace verbose phrases (”In order to do this, we then performed an analysis of…”) with terse equivalents (”We analysed…”).
- Remove every adjective that does not earn its place (”complex”, ”novel”, ”innovative”, ”comprehensive”).
- Combine adjacent sentences with semicolons or commas where appropriate.
- Move detail to the body. The Summary is not the place for derivations.

A good Summary at 400 words is usually better than the same content at 600.

What to Carry Into W3

Core explanation, worked reasoning, and application

Note: Class W2 Take-aways

1. The Summary disproportionately determines your score.
2. Use the four-block structure: Problem → Approach → Results → Insight.
3. The single most important sentence is the headline Results sentence. Make it impossible to miss.
4. Adjectives do not score points. *Numbers* score points.
5. Write the first Summary draft early; iterate at least three times.
6. Two writers always beat one writer on a high-stakes Summary.

What to Carry Into W3

In **W3: Assumptions, Validation, and Sensitivity Analysis** we attack the three sections that distinguish a thoughtful paper from a mathematical exercise. These are sections that average teams skim and great teams use to separate themselves.

Appendix A: Summary One-Page Template

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Appendix A: Summary template (1/2)

Summary

[Problem block: 2–4 sentences]

Context.
Specific question asked.
Why the problem is interesting/hard.

[Approach block: 3–5 sentences]

One-sentence strategy at a high level.
Model 1 (one sentence, named technique).
Model 2 (one sentence, refinement of Model 1).
Model 3 if any.
Validation source briefly mentioned.

Appendix A: Summary template (2/2)

Summary (continued)

[Results block: 3–5 sentences]

Headline numerical answer.
Supporting numerical findings.
One-line sensitivity note.

[Insight block: 2–3 sentences]

Why the result is trustworthy (validation/sensitivity).
Broader recommendation.

Appendix B: Headline-Sentence Templates

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Appendix B: Headline-Sentence Templates

- **Recommendation form:** *We recommend [doing X], using [N units of resource Y], at an estimated [cost/benefit Z].*
- **Estimate form:** *Our model predicts [quantity] = [value with units], with [error margin] across [scenario set].*
- **Comparison form:** *[Option A] outperforms [Option B] by [X%] on [metric], assuming [conditions].*
- **Conditional form:** *If [parameter] exceeds [threshold], we recommend [action 1]; otherwise [action 2].*

Use a template; modify; do not invent from scratch under competition pressure.

Appendix C: 25 Adjectives to Avoid in Summaries

Core explanation, worked reasoning, and application

Appendix C: 25 Adjectives to Avoid in Summaries

- novel
- innovative
- state-of-the-art
- cutting-edge
- revolutionary
- powerful
- robust (without qualifier)
- accurate (without number)
- efficient (without number)
- complex
- sophisticated
- comprehensive
- rigorous (without proof)
- thorough
- extensive
- insightful
- meaningful
- significant (in non-statistical sense)
- optimal (without proof)
- perfect
- ideal
- best-in-class
- world-class
- ground-breaking
- seamless

Appendix C: 25 Adjectives to Avoid in Summaries

Replace each with either (a) a number or (b) a verb. “Our novel approach…” → “We combine X and Y in a way not previously applied to Z.”