Writing Class 1: Paper Structure and Rubric Decoded

Mathematical Modeling Lecture

Mathematical Modeling

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Session Route

Launch

Question

Motivating context, key vocabulary, and the first modelling decision.

Build

Method

Worked example, diagram or table, calculation, and parameter meaning.

Test

Critique

Checkpoint practice, limitation check, and communication of results.

Columns are used where comparison helps; explanation slides stay clean and spacious.

Why This Class Exists

Core explanation, worked reasoning, and application

Why This Class Exists

By now you have spent months learning the mathematics of modeling: difference equations, regression, simulation, optimisation, dynamical systems. You are technically prepared. And yet, when judges sit down to evaluate your IMMC or HiMCM submission, they will not see your difference equation. They will see *your paper.*

A judge will spend approximately 5–10 minutes on the first read of your paper. In those minutes they decide whether to keep reading carefully, skim the rest, or move on. Within a 36-page submission, the first page (your **Summary**) will receive disproportionate attention. The mathematics behind it might be brilliant, but if the paper does not communicate that brilliance — clearly, in the structure judges expect — it will not be recognised.

This first class establishes:

- What the judges are scoring (their published rubric).
- The standard paper structure they expect.
- How to read a winning paper analytically, so you can imitate the moves that work.

The next four classes (W2–W5) drill into specific sections, language, and tools. Everything starts here.

Note: The Competition Reality

- **IMMC** (International Mathematical Modeling Challenge): teams of up to 4 students; 96 hours (Continental round) or 120 hours (International); papers up to ~ 25–30 pages; English required for International.
- **HiMCM** (High School Mathematical Contest in Modeling): teams of up to 4; 36 hours within a 14-day window; English; one of two problems (A or B).
- In both, the *paper itself* is what is judged — not your code, your simulations, or your conversations. Everything must reach the judge through the document.

The Standard Paper Structure

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The Standard Paper Structure

There is a near-universal structure for modeling competition papers. It is essentially the structure of a scientific research paper, adapted for the time and scope of a competition.

Method: Method: Standard IMMC/HiMCM Paper Skeleton

1. **Title page** — problem identifier, team number, date.
2. **Summary** (1 page; sometimes called Executive Summary or Abstract). *The single most important page.*
3. **Table of Contents** (optional but common).
4. **Introduction / Restatement of the Problem**
   - Background / motivation
   - Problem clarification
   - Our approach (high-level)
5. **Assumptions and Justifications**
6. **Variables, Parameters, Notation** (table form)
7. **Model Development**
   - Model 1 (often the simplest baseline)
   - Model 2 (refinement / extension)
   - Each with derivation, equations, intuition
8. **Solution / Results**

Method: Method: Standard IMMC/HiMCM Paper Skeleton

- Analytical results (if any)
- Numerical results (with figures and tables)

9. **Model Validation**
   - Consistency checks, limiting-case checks
   - Comparison with data or with literature
10. **Sensitivity Analysis**
    - How does the answer change when key parameters or assumptions change?
11. **Strengths and Weaknesses**
12. **Conclusion and Future Work**
13. **References** (formatted consistently)
14. **Appendices** (code, derivations, supplementary figures)

Warning: Warning: The Order is Not Optional

Judges expect the sections in approximately this order. They use it to navigate quickly: when they want to check ”did you sanity-check the model?”, they jump to *Validation*. If you put validation inside Model Development, or split it across three places, they will conclude you did not understand the conventions — which is a worse signal than not validating at all.

You may merge adjacent sections (e.g., *Strengths and Weaknesses* inside *Conclusion*) or rename them slightly, but **do not invent your own structure**.

Method: Method: Section-by-Section Purpose Statement

1. **Summary:** convince the judge to read further. State problem, approach, results, in one page.
2. **Introduction:** prove you understood the problem. Restate the problem in your own words; do not copy.
3. **Assumptions:** prove you thought about the real world. List what you assumed and why it is reasonable.
4. **Variables:** let the judge read equations without flipping back. Tabulate every symbol.
5. **Model Development:** prove you built something non-trivial. Show derivation, not just final equations.
6. **Solution:** answer the actual question that was asked. Numbers, with units, in context.
7. **Validation:** prove the model is not nonsense. Limiting cases, data comparison.
8. **Sensitivity Analysis:** prove the answer is robust. Vary parameters, report effect on conclusions.
9. **Strengths and Weaknesses:** prove self-awareness. Critique your own work honestly.
10. **Conclusion:** answer the question one more time, in plain language.

The Rubric: How Judges Actually Score You

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The Rubric: How Judges Actually Score You

Both IMMC and HiMCM publish (or have judges informally communicate) the criteria. The exact wording differs by year and contest, but the four pillars are stable.

Method: Method: The Four Pillars of Modeling Paper Evaluation

1. **Modeling Approach** — Is the modeling thought process clear? Are assumptions reasonable? Are the chosen methods appropriate? Is there originality or insight?
2. **Mathematical Sophistication** — Is the mathematics correct and at an appropriate level? Are the techniques used well? Does the team demonstrate command of the methods?
3. **Validation and Sensitivity** — Does the team check whether their model makes sense? Do they report how sensitive results are to inputs?
4. **Communication** — Is the paper clearly written, well organised, properly figured and cited? Is the Summary informative? Do the figures clarify?

Note: A Realistic Weighting (Approximate)

Although exact weights vary, a useful mental model:

Pillar	Approx. weight
Modeling Approach	25–30%
Mathematical Sophistication	25–30%
Validation & Sensitivity	15–20%
Communication (Writing)	25–30%

Communication is roughly equal to all the mathematics. This is the central insight that surprises most newcomers, and the reason for this entire Writing Class.

Method: Method: Things That Lose Points (and Why)

1. **Modeling Approach failures**
   - Skipping the simplest model and jumping straight to a fancy one.
   - Treating assumptions as filler (”we assume the world is real”) rather than as genuine modeling choices.
   - No clear connection between assumptions, model, and the question being answered.
2. **Mathematical Sophistication failures**
   - Equations appear without derivation or motivation.
   - Using a ”big” method (neural network, fuzzy logic, AHP) without justifying why a simpler method would not work.
   - Numerical errors in worked examples that propagate through the paper.
3. **Validation & Sensitivity failures**
   - Validation section omitted entirely (alarmingly common).
   - ”Sensitivity analysis” that is one paragraph of words with no numbers.
   - Reporting only that the result is ”robust” without saying *to what* and *by how much*.
4. **Communication failures**

Method: Method: Things That Lose Points (and Why)

- Summary that says what you *did* but not what you *found*.
- Equations without numbered references; figures without captions; tables without units.
- Walls of text with no headings; pages of code in the body of the paper.
- Inconsistent notation (using r for radius in one section and growth rate in the next).

Anatomy of a Winning Paper

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Anatomy of a Winning Paper

Let us walk through how the four pillars apply, using a concrete (illustrative) example. Imagine the problem: *model the throughput of a single-server queue at a coffee shop and recommend a staffing strategy.*

Worked Example: Example 1: A Strong Modeling Approach Section (Sketch)

We begin with the simplest defensible model and progressively refine.

Model 1 (baseline): Assume customers arrive deterministically every 5 minutes and service takes exactly 4 minutes. Then no queue ever forms, and the throughput is 12 customers/hour. This baseline gives a sanity-check ceiling.
Model 2 (stochastic arrivals): Customers arrive as a Poisson process with rate λ. Service times remain deterministic at 4 minutes. Under this assumption, queues form occasionally; we use simulation to estimate average wait.
Model 3 (full M/M/1): Service times exponential with rate $\mu = 1/4$ per minute. Now both standard queuing theory results and simulation apply, and the two can validate each other.

Note: Why this Modeling Approach is strong

- Starts simple; each model is motivated by what the previous missed.
- Each model is a *stepping stone*, not a side trip.
- Validation is baked in: Model 1 gives a sanity ceiling; Models 2 and 3 cross-check.

Worked Example: Example 2: A Strong Assumptions Section (Sketch)

Assumption 1. Customer arrivals follow a Poisson process with rate λ.
Justification: Coffee shops in similar settings have been modeled this way (Larson & Odoni, 1981), and the assumption of independent arrivals is reasonable for unaffiliated customers at random times.
Effect on model: Allows use of standard queuing results and inter-arrival time generation by inverse-transform sampling.
Assumption 2. Service times are exponentially distributed with rate μ.
Justification: Choice driven by the memorylessness property; arguably less realistic than a deterministic time but vastly easier to analyse.
Effect: The system becomes an $M/M/1$ queue. We will check the result against simulation with deterministic service in §6.
Assumption 3. The shop operates from 7:00 to 10:00 with constant λ.
Justification: Simplifies analysis at the cost of accuracy; in reality, λ peaks at 8:00.
Effect: Our results give an average estimate; peak conditions may need separate treatment.

Note: Why this Assumptions section is strong

Each assumption has three parts:

1. **What** is assumed.
2. **Why** it is reasonable (citation, simplification, or domain knowledge).
3. **What effect** it has on the model.

Many teams write only (1). A judge can tell instantly.

Worked Example: Example 3: A Strong Sensitivity Analysis (Sketch)

We test sensitivity of the predicted average wait time to three parameters:

- Arrival rate λ: vary from $\pm 20\%$ of nominal.
- Service rate μ: vary $\pm 20\%$.
- Assumption of exponential service: replace with deterministic service.

Findings.

Scenario	λ	μ	Avg wait (min)	Change
Baseline	12/hr	15/hr	4.00	—
Higher λ	14.4/hr	15/hr	9.60	+140%
Lower μ	12/hr	12/hr	∞	system unstable
Higher μ	12/hr	18/hr	1.33	-67%
Deterministic service	12/hr	15/hr	2.00	-50%

Worked Example: Example 3: A Strong Sensitivity Analysis (Sketch)

Interpretation. The model is highly sensitive to λ and μ near the stability boundary $\lambda \approx \mu$. The deterministic-service comparison shows our exponential-service assumption *overstates* the queue by roughly 50%, so our recommendation will be conservative.

Note: Why this Sensitivity Analysis is strong

- Identifies *which* parameters or assumptions matter.
- Reports actual numbers, not just words.
- Connects findings back to the conclusion ("our recommendation is conservative").
- Includes a structural sensitivity (the exponential vs deterministic comparison) — not just numerical perturbation.

Reading a Real Outstanding Paper

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Reading a Real Outstanding Paper

The single best preparation for writing well is to read good papers analytically. COMAP publishes **Outstanding Papers** from HiMCM and MCM annually; IMMC publishes its top-scoring papers as well. Reading three or four of these — not for content but for structure — is more valuable than reading any guide, including this one.

Method: Method: How to Read an Outstanding Paper

Allocate 90 minutes per paper. Read with a notebook and answer these questions:

1. **Summary** (5 min): Does it tell you the problem, the approach, the result, and the key insight, in roughly that order? How long is each part?
2. **Section Headers** (5 min): What is their section structure? How does it differ from the skeleton in §2 of this class?
3. **Assumptions** (10 min): How many do they list? How are they justified? Do they explicitly call out the effect of each assumption?
4. **Model Development** (20 min): How do they introduce equations? Always with words first? With derivations or just statements? How do they use figures?
5. **Results** (15 min): Where do the numerical answers appear? Are they highlighted (boxed, bolded, or in a result table)?
6. **Validation & Sensitivity** (15 min): How much space is allocated? What types of checks did they perform?
7. **Writing Style** (10 min): Average paragraph length? Active or passive voice? Use of ”we”? Tone?

Method: Method: How to Read an Outstanding Paper

8. **Figures & Tables** (10 min): How many? Captioned? Referenced from the text? Use of colour, fonts, captions?

Build a one-page **template extract** based on your findings, then compare across multiple papers to find the patterns.

Note: Where to Find Outstanding Papers

- **COMAP.** The COMAP website publishes Outstanding papers for HiMCM and MCM each year. They are freely accessible.
- **UMAP Journal.** The *UMAP Journal* prints selected Outstanding papers with judges' commentaries — the commentaries are particularly valuable.
- **IMMC Chinese region.** The IMMC China Hong Kong page publishes lists and sometimes selected papers from the Continental round.
- **Caution.** Older papers (pre-2010) have different writing conventions; focus on the last 5 years.

Common Mistakes Made by First-Time Teams

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Warning: Warning: The Mistakes That Most Hurt First-Time Teams

Read these now. Re-read them the day before submission.

1. **Writing the Summary last (and rushing it).** The Summary determines the first impression; treat it like the second-most-important section, not an afterthought.
2. **Copying the problem statement instead of restating it.** Judges read this immediately and conclude you did not engage. Rewrite the problem in your own words.
3. **Listing assumptions without effects.** ”We assume the Earth is flat” is not an assumption; an assumption needs a reason and a consequence.
4. **Hiding the answer.** The numerical recommendation or final result must be **boxed, bolded, or otherwise impossible to miss**. Many strong papers fail because the judge cannot find the answer.
5. **No connection between models.** If you have three models, explain how each relates to the others. Are they alternatives? Refinements? Are results consistent?
6. **Equations without numbers.** Number every important equation. Reference them by number from the text.
7. **Figures without captions.** Every figure needs a caption that stands on its own; the reader should not need the body text to understand the figure.

Warning: Warning: The Mistakes That Most Hurt First-Time Teams

8. **Code in the body.** Code goes in the Appendix. The body text describes *what* the code does and *why*; the code itself is supporting material.
9. **Skipping Sensitivity Analysis ”because we ran out of time”.** Better to truncate Model Development than to skip Sensitivity. Judges flag the omission immediately.
10. **No references.** A paper with zero citations signals isolation from the literature. Even 3–4 citations to standard textbooks (Giordano; Hillier & Lieberman; Bertsekas) is dramatically better than none.

Workshop Exercises

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Exercises: Exercise W1.1: Diagnose a Section

Below is a one-paragraph Assumptions section. Identify all the problems and rewrite it in the three-part format from Example 2.

> *We assume that the population is constant and there is no migration. We also assume the disease is contagious and people get sick from each other. We assume the recovery rate is constant and people can recover. There is no death due to other causes.*

Exercises: Exercise W1.2: Identify the Pillar Failures

For each of the following short excerpts from imagined papers, identify which of the four pillars (Modeling, Math, Validation, Communication) is being undermined and explain why.

1. "We applied a fuzzy-logic neural network with 24 hidden layers to determine the optimal car park entry fee."
2. "Our model predicts the answer is $\$3.47$ per hour. This concludes our analysis."
3. "Eq. 12 gives the average waiting time, where everything is as defined above."
4. "After running our simulation, we conclude that the queue length is robust to all parameter changes."
5. "We assume the price elasticity is -0.5. We assume the population grows. We assume the year has 365 days."

Exercises: Exercise W1.3: Skeleton Practice

Take any of the following short modeling questions. For each, sketch a section-by-section outline (1–2 sentences per section) following the standard skeleton from §2. *Do not solve the problem.* Just structure how you would present a solution.

1. How should a school schedule its bus routes to minimise total travel time?
2. How long should a stoplight stay green at a single-intersection city centre?
3. What is the carrying capacity of a small island for a deer population?

Exercises: Exercise W1.4: Outstanding Paper Reading

Choose one Outstanding paper from HiMCM (any year 2020 or later) from the COMAP website. Spend 90 minutes reading it analytically per Method 5.1. Produce:

- A bullet-list of the section headers (with subsections).
- Counts: number of equations, number of figures, number of tables, number of references.
- A one-paragraph summary of how the team handled Sensitivity Analysis.
- A one-paragraph judgment: what one move from this paper will you copy in your own?

What to Carry Into W2

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Note: Class W1 Take-aways

1. Papers are scored on four pillars; **Communication is roughly** 25–30% — equal to all the mathematics.
2. The standard structure is rigid; do not innovate at the section-organisation level.
3. Every section has a **specific purpose**; if you cannot state in one sentence what a paragraph contributes, cut it.
4. Assumptions need three parts: *what*, *why reasonable*, *effect on model*.
5. Sensitivity must include numbers, not just words.
6. Reading 3–4 Outstanding papers is the highest-leverage preparation you can do.

What to Carry Into W2

In **W2: The Summary** we attack the single most important page of your paper: the Summary. We will look at what a great Summary looks like, decompose its standard four-block structure, and write three versions of a Summary against the same problem.

Appendix A: One-Page Cheat Sheet

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Appendix A: One-Page Cheat Sheet

Section	One-sentence purpose
Summary	Convince the judge your paper is worth reading.
Introduction	Show you understood the question.
Assumptions	Show you thought about the real world.
Variables	Let the judge read equations smoothly.
Model	Show you built something non-trivial.
Solution	Answer the question with numbers, units, context.
Validation	Show the model is not nonsense.
Sensitivity	Show the answer is robust.
Strengths/Weaknesses	Show self-awareness.
Conclusion	Answer one more time, in plain language.
References	Show grounding in the literature.
Appendix	Code, derivations, supplementary figures.

Appendix B: Pre-Submission Checklist

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Appendix B: Pre-Submission Checklist

- Summary fits on **one page** and follows a clear four-block structure (Problem $\rightarrow$ Approach $\rightarrow$ Results $\rightarrow$ Insight).
- Every assumption has *what*, *why*, *effect*.
- Every variable used in an equation is in the Variables table.
- Every equation is numbered.
- Every figure has a caption that stands alone.
- Every table has a title and units in column headers.
- Validation section is present and includes at least *one* concrete check (limiting case, data comparison, or alternative-model comparison).
- Sensitivity Analysis includes a *table* with parameter changes and result changes in numbers.
- Strengths and Weaknesses are both present; at least two of each.
- Conclusion answers the actual question asked.
- References use a consistent format (APA, IEEE, or similar). At least 4 entries.
- Code is in the Appendix, not in the body.
- Page numbers visible, team number on every page (if required by contest).