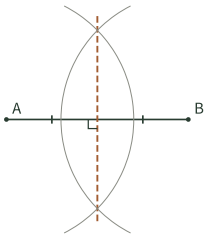
Constructing symmetry axes with ruler and compass — Teacher answers

Year 8, upper volume • text pp. 33–34 • foundation lesson 17

Use a ruler and compass; keep arcs visible. Show the solution axis only in the teacher answer. Check midpoint and perpendicularity separately. This lesson constructs axes, not later coordinate rules or whole reflected polygons.

Worked example 1

A and B are a pair of corresponding points. Construct their symmetry axis with ruler and compass. Keep the arc intersections.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

The full line through the midpoint of AB, perpendicular to AB. Construct it with two equal-radius arc intersections.

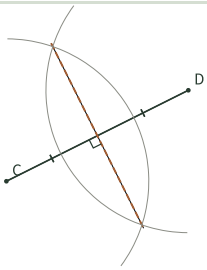
1. Choose one compass radius greater than half the drawn segment AB.
2. With centres A and B, draw arcs on both sides to obtain two distinct intersections X and Y.
3. Draw the full line XY. It is the perpendicular bisector of AB.
4. Mark the segment midpoint and the right angle, then check that the two endpoints exchange under reflection.

- Show equal-radius arcs with two distinct intersections.
- Draw a full line through the midpoint and perpendicular to the segment.
- Retain construction traces and check the corresponding endpoints.

Guided attempt 2

C and D are distinct corresponding points on the slanted segment. Complete the construction using the scaffold, without estimating the axis by eye.

1. Choose radius greater than ___ of CD.
2. Use centres ___ and ___ with the same radius to obtain X and Y.
3. Draw line ___; check it is ___ to CD and passes through its ___.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

The perpendicular bisector of CD, formed by the two equal-radius arc intersections.

1. Take a common radius greater than half CD.
2. Draw arcs from C and D on both sides, meeting at X and Y.
3. Line XY passes through the midpoint and is perpendicular to CD.
4. Extend the line and leave the arcs as evidence of the construction.

- Use equal-radius arcs and draw the perpendicular bisector as a full line.
- Keep traces rather than relying on apparent symmetry.

Prerequisite 3

Which two checks certify that a line is the axis exchanging distinct A and A′?

A. It is perpendicular to AA′ somewhere.
B. It only passes through the midpoint.
C. It passes through the midpoint of AA′ and is perpendicular to AA′.
D. It passes through A and A′.

Reference answer

Both midpoint and perpendicularity are required.

1. A reflection axis bisects the segment joining distinct corresponding points.
2. It is also perpendicular to that joining segment.

- State both conditions together.

Prerequisite 4

To obtain two distinct intersections of equal-radius circles centred at A and B, how should the radius compare with half AB?

A. It must equal half AB.
B. It must be smaller than half AB.
C. It must be zero.
D. It must be greater than half AB.

Reference answer

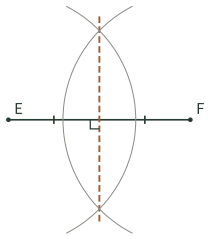
Greater than half AB; equality gives only one touching point.

1. With equal radii r, two intersections require AB<2r.
2. Hence r>AB/2. At equality the circles touch at the midpoint.

- Use a strict greater-than condition and explain why equality is insufficient.

Independent practice 5

E and F are corresponding points. Construct their full symmetry axis with ruler and compass.



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Reference answer

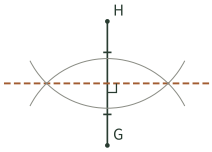
The full line through the midpoint of EF, perpendicular to EF. Construct it with two equal-radius arc intersections.

1. Choose one compass radius greater than half the drawn segment EF.
2. With centres E and F, draw arcs on both sides to obtain two distinct intersections X and Y.
3. Draw the full line XY. It is the perpendicular bisector of EF.
4. Mark the segment midpoint and the right angle, then check that the two endpoints exchange under reflection.

- Show equal-radius arcs with two distinct intersections.
- Draw a full line through the midpoint and perpendicular to the segment.
- Retain construction traces and check the corresponding endpoints.

Independent practice 6

G and H are corresponding points on the vertical segment. Construct their symmetry axis and mark the right angle.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

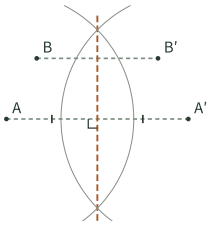
The full line through the midpoint of GH, perpendicular to GH. Construct it with two equal-radius arc intersections.

1. Choose one compass radius greater than half the drawn segment GH.
2. With centres G and H, draw arcs on both sides to obtain two distinct intersections X and Y.
3. Draw the full line XY. It is the perpendicular bisector of GH.
4. Mark the segment midpoint and the right angle, then check that the two endpoints exchange under reflection.

- Show equal-radius arcs with two distinct intersections.
- Draw a full line through the midpoint and perpendicular to the segment.
- Retain construction traces and check the corresponding endpoints.

Independent practice 7

A corresponds to A′ and B corresponds to B′. Construct one line that exchanges both pairs, and show how the second pair checks your first construction.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

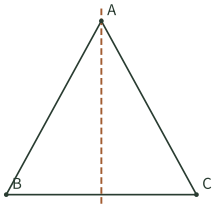
The common perpendicular bisector of AA′ and BB′. The two constructed perpendicular bisectors coincide.

1. Construct the perpendicular bisector of AA′ with equal-radius arcs.
2. Check that the line also passes through the midpoint of BB′ and is perpendicular to BB′.
3. Alternatively construct the second perpendicular bisector; it coincides with the first.
4. Extend this one common line; do not count it twice.

- Construct a valid axis and explicitly verify the other pair.

Independent practice 8

In the non-equilateral isosceles triangle, AB=AC. Use ruler and compass to construct the symmetry axis. Explain why the line you construct passes through A.



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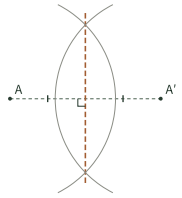
Construct the perpendicular bisector of BC. Since AB=AC, the converse puts A on that line; it is the triangle’ s only axis.

1. Use equal-radius arcs from B and C to construct the perpendicular bisector of BC.
2. AB=AC means A is equidistant from B and C, so A lies on the constructed line.
3. Reflection fixes A and exchanges B and C; the entire triangle matches.

- Retain compass traces for the base perpendicular bisector.
- Use equal endpoint distances to justify that the axis passes through A.

Exit question 9

A pupil joins distinct corresponding points A and A′ and labels the joining line “axis”. Explain the error and state the line that should be constructed.



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Reference answer

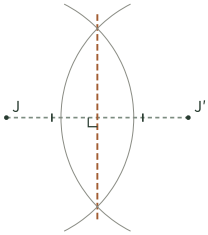
The joining line would fix A and A′ instead of exchanging them. Construct its perpendicular bisector.

1. Every point on a reflection axis remains fixed.
2. A and A′ are distinct points to be exchanged, so their joining line cannot be the axis.
3. The correct line passes through their midpoint at a right angle.

- Explain the fixed-point error and name the perpendicular bisector.

Exit question 10

J and J′ are a corresponding pair. Draw their full axis, and mark both the midpoint and right-angle checks. Keep any compass traces used.



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Reference answer

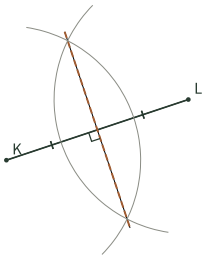
The line through the midpoint of JJ′ perpendicular to JJ′, extended in both directions.

1. Construct or verify the midpoint of JJ′ with equal-radius arcs.
2. The line through the two arc intersections is perpendicular to JJ′.
3. Extend the line and mark both required checks.

- Show a full perpendicular bisector with midpoint and right-angle evidence.

Further practice 11

K and L are corresponding points on a slanted segment. Construct the axis without measuring its angle.



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Reference answer

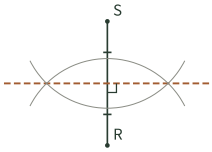
The full line through the midpoint of KL, perpendicular to KL. Construct it with two equal-radius arc intersections.

1. Choose one compass radius greater than half the drawn segment KL.
2. With centres K and L, draw arcs on both sides to obtain two distinct intersections X and Y.
3. Draw the full line XY. It is the perpendicular bisector of KL.
4. Mark the segment midpoint and the right angle, then check that the two endpoints exchange under reflection.

- Show equal-radius arcs with two distinct intersections.
- Draw a full line through the midpoint and perpendicular to the segment.
- Retain construction traces and check the corresponding endpoints.

Further practice 12

R and S are corresponding points. Construct the axis, preserving the same compass radius from both centres.



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Reference answer

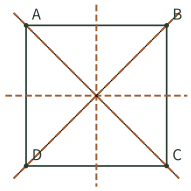
The full line through the midpoint of RS, perpendicular to RS. Construct it with two equal-radius arc intersections.

1. Choose one compass radius greater than half the drawn segment RS.
2. With centres R and S, draw arcs on both sides to obtain two distinct intersections X and Y.
3. Draw the full line XY. It is the perpendicular bisector of RS.
4. Mark the segment midpoint and the right angle, then check that the two endpoints exchange under reflection.

- Show equal-radius arcs with two distinct intersections.
- Draw a full line through the midpoint and perpendicular to the segment.
- Retain construction traces and check the corresponding endpoints.

Further practice 13

Use ruler and compass to draw all axes of a square. Include both side-midpoint axes and vertex axes.



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Reference answer

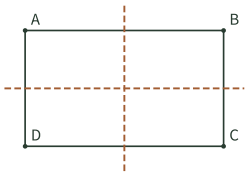
Four lines: the perpendicular bisectors of two adjacent sides and the two diagonals.

1. Construct the perpendicular bisectors of two adjacent sides with equal-radius arcs.
2. Draw the two diagonals using the ruler.
3. Extend the four different lines and verify the whole-square reflection test.

- List or draw all valid full lines without counting one line twice.
- Explain the fold test and exclude invalid candidates.
- Retain traces for the perpendicular-bisector constructions.

Further practice 14

Use ruler and compass to construct all axes of a non-square rectangle. Give a reason for leaving out the diagonals.



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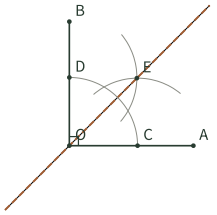
Two lines: the perpendicular bisectors of two adjacent sides. Diagonal folds fail because the adjacent side lengths differ.

1. Construct the perpendicular bisectors of two adjacent sides.
2. They pass through the rectangle centre and the opposite side midpoints.
3. Check the two full axes; reject diagonal candidates using unequal adjacent sides.

- List or draw all valid full lines without counting one line twice.
- Explain the fold test and exclude invalid candidates.
- Show compass traces for both midpoint axes.

Extension 15

OA and OB are two rays forming a right angle. Construct the symmetry axis of this two-ray figure using ruler and compass, and explain which ray each side reflects to.



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Reference answer

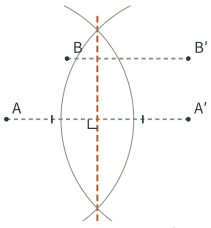
The full line through O along the internal angle bisector; it exchanges OA and OB.

1. Draw an arc centred at O meeting the rays at C and D, so OC=OD.
2. Use equal-radius arcs centred at C and D to obtain an intersection E inside the angle, different from O.
3. Draw the full line OE; the equal-side construction bisects the angle.
4. Reflection in this line fixes O and exchanges the two rays.

- Use equal-radius arc construction and identify the internal-bisector direction.
- Draw a full axis and explain the exchanged rays.

Extension 16

A corresponds to A′ and B corresponds to B′. Can one line exchange both supplied pairs? Use ruler and compass to compare their required axes, rather than choosing a line halfway between them.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

No. The two required perpendicular bisectors are parallel but distinct; a common axis cannot satisfy both pairs.

1. Construct the perpendicular bisector of AA′; it is the unique axis for that distinct pair.
2. Construct the perpendicular bisector of BB′.
3. The two lines are parallel and do not coincide, so no single line works for both pairs.
4. An averaged line would fail at least one pair and is not a substitute.

- Construct or identify both perpendicular bisectors.
- Explain why distinct required axes rule out a common axis.