Squares and inclusive quadrilateral classification — Teacher answers

Year 8, lower volume • text pp. 45–48 • textbook lessons 23–24

Use the two prerequisite checks to choose support. Project the worked and guided tasks, then collect the eight paper core questions. Follow-ups and extensions are optional; accept other complete valid methods. This page does not save learners' paper answers.

Ordinary route: prerequisites 4, short inquiry 5, worked/guided 8, independent 15, correction 5, exit 3 minutes. Advanced route: inquiry 18, organise 4, selected practice 13, exit 5 minutes.

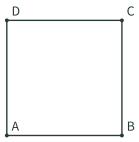
Prerequisites: Parallelograms, rectangles, rhombi and their criteria.

Ordinary route: use the first fixed case for the five-minute demonstration. Advanced route: complete the full investigation.

1. 3′ Predict the strongest conclusion from three cards: parallelogram plus equal diagonals; plus perpendicular diagonals; plus both.
2. 8′ Sort the cards using a non-square 4-by-3 rectangle, a non-square rhombus with diagonals 6 and 8, and a square as examples. Label which conditions each actually satisfies. Keep a square inside the rectangle, rhombus and parallelogram families.
3. 5′ Explain why either diagonal condition alone is insufficient for a square and why both suffice for a parallelogram. Give a counterexample to the stronger unsupported claim.
4. 2′ Individually complete: a rhombus with one right angle is a _____, because _____.

Worked example 1

ABCD is a rectangle with AB=BC. Prove it is a square.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

The rectangle gives right angles and equal opposite sides. AB=BC then makes all four sides equal, so it is a square.

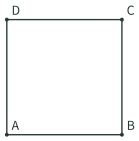
1. Use the rectangle premise for the right angles.
2. Use equal opposite sides with AB=BC to obtain four equal sides.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Guided attempt 2

ABCD is a rhombus with one right angle. Explain why it is a square.

1. Rhombus ⇒ four _____ sides and two pairs of _____ opposite sides.
2. One angle is 90°; its adjacent angle is $180^\circ - \square = \square$.
3. Four equal sides + four right angles ⇒ _____.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

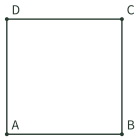
It has four equal sides and, as a parallelogram with a right angle, four right angles; it is a square.

1. A rhombus is a parallelogram with equal sides.
2. One right angle gives the rectangle condition; combine both families.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Prerequisite 3

Does a square count as a rectangle under the inclusive definition? Explain.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

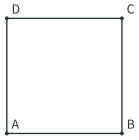
Yes; it satisfies the rectangle definition.

1. Four equal sides do not remove its right angles.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Prerequisite 4

Does a square count as a rhombus? Explain.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

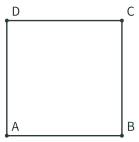
Yes; all four sides are equal.

1. The extra right angles do not remove the equal-side property.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 5

A rectangle has perpendicular diagonals. State and justify its strongest classification.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

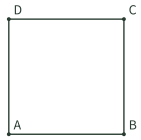
A square: perpendicular diagonals make this parallelogram a rhombus; it is already a rectangle.

1. Retain the rectangle premise, then use the rhombus criterion.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 6

A rhombus has equal diagonals. State and justify its strongest classification.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

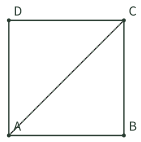
A square: equal diagonals make this parallelogram a rectangle; it is already a rhombus.

1. Use the equal-diagonal criterion together with the parallelogram premise inherited from the rhombus.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 7

A square has side 4 cm. Find its diagonal exactly and name the right triangle used.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

$4\sqrt{2}$ cm.

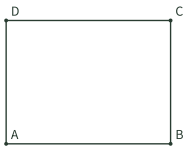
$$4\sqrt{2}$$

1. Its diagonal forms a right triangle with two 4 cm legs.
2. $AC^2 = 4^2 + 4^2 = 32$; $AC = 4\sqrt{2}$ cm.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 8

A rectangle has adjacent sides 8 cm and 6 cm. A learner calls it a square because the drawing looks nearly equal. Correct the claim.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

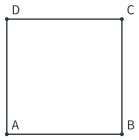
It is a rectangle, not a square, because adjacent side lengths differ.

1. Use the stated lengths, not measured appearance.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Exit question 9

Is every square a parallelogram? Explain.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

Yes; both opposite-side pairs are parallel.

1. Being more specialised does not exclude the broader family.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Exit question 10

Must every rectangle be a square? Give a counterexample.



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Reference answer

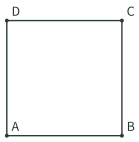
No. A rectangle with sides 8 cm and 6 cm is a counterexample.

1. Right angles do not force adjacent sides to be equal.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 11

A square has $AC = 4\sqrt{2}$ cm. Find BD.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

$4\sqrt{2}$ cm.

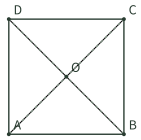
$$4\sqrt{2}$$

1. A square is a rectangle, so its diagonals are equal.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 12

The diagonals of square ABCD meet at O. Find $\angle AOB$ and name the inherited property.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

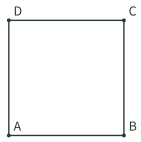
90°; a square is a rhombus, whose diagonals are perpendicular.

1. Use the rhombus diagonal property.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 13

A square has side 4 cm. Find its area and perimeter, keeping the units distinct.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

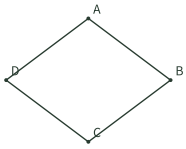
Area 16 cm^2; perimeter 16 cm. The numerical values agree but their units differ.

1. Area=$4\times4=16$ cm^2; perimeter=$4\times4=16$ cm.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 14

Four equal sides are given in a simple quadrilateral. Is that sufficient for a square?



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Reference answer

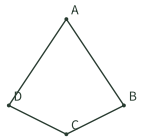
No; the shown non-square rhombus has four equal sides but no right angles.

1. A square also requires the right-angle condition.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Extension 15

A general quadrilateral has equal, perpendicular diagonals. Must it be a square? Test the shown kite and identify a missing condition.



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Reference answer

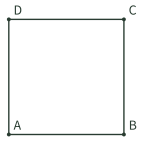
No. Both diagonals are 4 units and perpendicular, but one does not bisect the other. The parallelogram condition is missing.

1. The vertical diagonal is split into lengths 3 and 1.
2. Equal and perpendicular without bisection do not give a square.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Extension 16

A parallelogram has equal and perpendicular diagonals. Prove it is a square by using two different criteria.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

Equal diagonals give a rectangle; perpendicular diagonals give a rhombus. The same quadrilateral belongs to both families and is a square.

1. Apply each criterion with the original parallelogram premise.
2. Combine right angles and four equal sides.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.