# Rectangles and rhombi: diagonal conditions and converses — Teacher answers

Year 8, lower volume • text pp. 37–44 • textbook lessons 19–22

Use the two prerequisite checks to choose support. Project the worked and guided tasks, then collect the eight paper core questions. Follow-ups and extensions are optional; accept other complete valid methods. This page does not save learners' paper answers.

Ordinary route: prerequisites 4, short inquiry 5, worked/guided 8, independent 15, correction 5, exit 3 minutes. Advanced route: inquiry 18, organise 4, selected practice 13, exit 5 minutes.

**Prerequisites:** Parallelogram properties, Pythagoras and congruence.

Ordinary route: use the first fixed case for the five-minute demonstration. Advanced route: complete the full investigation.

1. 3′ Predict which diagonal properties a non-square rectangle and a non-square rhombus share.
2. 8′ On paper compare a 4-by-3 rectangle (diagonals 5) with the rhombus (0,0),(3,4),(6,0),(3,−4) (diagonals 6 and 8). Record bisection, equal lengths and perpendicularity separately. Use the stated shapes as premises, not their apparent outlines.
3. 5′ Identify the shared bisection property and the differing equal/perpendicular properties. Keep the parallelogram premise when using diagonal criteria.
4. 2′ Individually judge whether perpendicular diagonals alone force an arbitrary quadrilateral to be a rhombus; give a missing condition.

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**Worked example** 1

ABCD is a rectangle with AC=10 cm. Find BD and name the special rectangle property.




Geometry diagram; not drawn to scale except for construction tasks.

### Reference answer

$BD = 10$ cm.

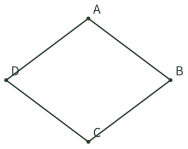
1. The diagonals of a rectangle are equal.
2. BD=AC=10 cm.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

**Guided attempt** 2

ABCD is a rhombus with AB=5 cm. Find CD and explain why no diagonal calculation is needed.

1. A rhombus has ____ equal sides.
2. $AB = BC = CD = DA = \square$ cm.
3. Therefore CD=____ cm; the information needed is a side length, not a diagonal length.




Geometry diagram; not drawn to scale except for construction tasks.

### Reference answer

$CD = 5$ cm.

1. Every side of a rhombus is equal.
2. CD=AB=5 cm.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

**Prerequisite** 3

A parallelogram has one right angle. Which special type must it be?




Geometry diagram; not drawn to scale except for construction tasks.

**Reference answer**

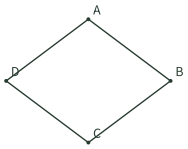
A rectangle.

1. Use the one-right-angle criterion with the parallelogram premise.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

**Prerequisite** 4

A parallelogram has AB=BC. Which special type follows?




Geometry diagram; not drawn to scale except for construction tasks.

**Reference answer**

A rhombus.

1. Equal adjacent sides plus equal opposite sides give all four sides equal.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

**Independent practice** 5

ABCD is a parallelogram and AC=BD. State the strongest classification justified by these conditions.




Geometry diagram; not drawn to scale except for construction tasks.

**Reference answer**

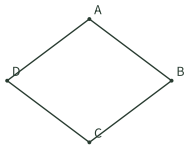
A rectangle; equal diagonals alone do not also guarantee a square.

1. Use equal diagonals with the known parallelogram.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

**Independent practice** 6

ABCD is a parallelogram and AC⊥BD. Which special type follows? Explain the required premise.




Geometry diagram; not drawn to scale except for construction tasks.

### Reference answer

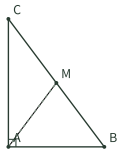
A rhombus. Perpendicular diagonals are used together with the parallelogram premise.

1. A general quadrilateral with perpendicular diagonals need not have equal sides.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

**Independent practice** 7

∠A=90° in △ABC, BC=10 cm, and M is the midpoint of BC. Find AM.




Geometry diagram; not drawn to scale except for construction tasks.

### Reference answer

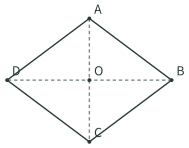
$AM = 5$ cm.

1. The median to the hypotenuse of a right triangle equals half the hypotenuse.
2. AM=10/2=5 cm.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

**Independent practice** 8

A rhombus has diagonals AC=6 cm and BD=8 cm, meeting at O. Find its side AB, explaining both diagonal properties used.




Geometry diagram; not drawn to scale except for construction tasks.

### Reference answer

AB=5 cm. The diagonals are perpendicular and bisect each other, giving legs 3 and 4.

1. AO=3 and BO=4, with ∠AOB=90°.
2. $AB^2=3^2+4^2=25$, so AB=5 cm.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

**Exit question** 9

Is every rectangle a parallelogram? Explain using the inclusive definition.




Geometry diagram; not drawn to scale except for construction tasks.

**Reference answer**

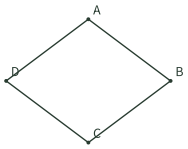
Yes. A rectangle is a special parallelogram.

1. Its opposite sides remain parallel even with the extra right-angle condition.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

**Exit question** 10

Must every rhombus be a rectangle? Use the shown non-square rhombus.




Geometry diagram; not drawn to scale except for construction tasks.

**Reference answer**

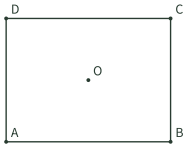
No. Equal sides do not force a right angle.

1. This rhombus has perpendicular diagonals but unequal diagonal lengths.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

**Further practice** 11

A rectangle has AC=10 cm and diagonals meeting at O. Find BO.




Geometry diagram; not drawn to scale except for construction tasks.

**Reference answer**

$BO = 5$ cm.

1. BD=AC=10 and O bisects BD.
2. BO=5 cm.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

**Further practice** 12

A general quadrilateral has equal diagonals. Must it be a rectangle? Explain with the shown trapezium.




Geometry diagram; not drawn to scale except for construction tasks.

**Reference answer**

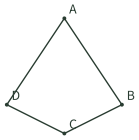
No. The parallelogram premise is missing.

1. The equal-diagonal isosceles trapezium has only one pair of parallel opposite sides.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

**Further practice** 13

A general quadrilateral has perpendicular diagonals. Must it be a rhombus? Give a reason from the shown kite.




Geometry diagram; not drawn to scale except for construction tasks.

**Reference answer**

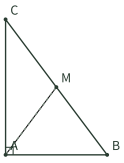
No. The two upper sides and two lower sides have different lengths.

1. Perpendicularity alone does not guarantee diagonal bisection or four equal sides.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

**Further practice** 14

In right $\triangle$ABC, M is the midpoint of hypotenuse BC and AM=10 cm. Find BC.




Geometry diagram; not drawn to scale except for construction tasks.

**Reference answer**

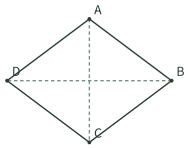
$BC = 20$ cm.

1. AM=BC/2, so BC=20 cm.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

**Extension** 15

A rhombus has diagonals 6 cm and 8 cm. Derive its area by splitting it into four right triangles.




Geometry diagram; not drawn to scale except for construction tasks.

### Reference answer

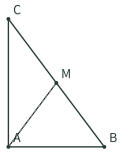
24 cm². Each triangle has legs 3 and 4, area 6, and there are four.

1. The diagonals are perpendicular and bisect each other.
2. Total area=4×(3×4/2)=24 cm².

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

**Extension** 16

In a nondegenerate triangle ABC, M is the midpoint of BC and AM=BM. Show that ∠BAC is a right angle, using angles in the two isosceles triangles.




Geometry diagram; not drawn to scale except for construction tasks.

### Reference answer

AM=BM=CM. Let ∠B=x and ∠C=y. The angle at A is x+y, so 2(x+y)=180° and ∠A=90°.

1. M is the midpoint inside BC, so $BM = CM$; together with the given $AM = BM$, this gives $AM = BM = CM$.
2. By the two isosceles triangles, let $\alpha = \angle BAM = \angle ABM$ and $\beta = \angle MAC = \angle ACM$.
3. Since M lies between B and C in a nondegenerate triangle, $\angle A = \alpha + \beta$, $\angle B = \alpha$, and $\angle C = \beta$.
4. The triangle angle sum gives $(\alpha + \beta) + \alpha + \beta = 180^\circ$, hence $2(\alpha + \beta) = 180^\circ$.
5. Therefore $\angle BAC = \alpha + \beta = 90^\circ$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.