Rational operations, integer powers and scientific notation — Teacher answers

Year 8, upper volume • text pp. 83–96 • foundation lessons 42–48

Use the worked model and guided attempt selectively, then the eight core questions. Keep further practice and extensions optional. Ask for a reason and an original-equation or expansion check.

Worked example 1

Simplify and retain every restriction from the original expression.

$$\frac{1}{x-1}+\frac{2}{x+1}$$

Reference answer

(3x-1)/(x²-1); x≠-1, 1.

$$\frac{3x-1}{x^2-1}$$

1. First record x≠-1, 1 from the original denominators and any divisor.
2. Use a common denominator and multiply the entire numerator by the required factor.
3. (1)/(x-1)+(2)/(x+1) = ((x+1)+2(x-1))/((x-1)(x+1)).
4. Therefore (3x-1)/(x²-1), with x≠-1, 1 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Guided attempt 2

Simplify and retain every restriction from the original expression.

$$\frac{x^2-4}{x+1}\div\frac{x-2}{x+1}$$

1. Original denominators exclude ___; the divisor also excludes ___.
2. Replace ÷ by × the reciprocal, factor, then cancel ___.

Reference answer

x+2; x≠-1, 2.

$$x+2$$

1. First record x≠-1, 2 from the original denominators and any divisor.
2. Replace division by multiplication by the reciprocal; the original divisor must remain nonzero.
3. (x²-4)/(x+1) ÷ (x-2)/(x+1) = ((x-2)(x+2))/(x+1) × (x+1)/(x-2).
4. Therefore x+2, with x≠-1, 2 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Prerequisite 3

Evaluate 2^{-3}. Explain the negative exponent.

$$2^{-3}$$

Reference answer

1/8.

$$\frac{1}{8}$$

1. A negative exponent means a reciprocal, not a negative value.
2. $2^{-3}=1/2^3=1/8$.

- Use the stated relationship and show the calculation.
- State the result with its unit where appropriate.

Prerequisite 4

Simplify and retain every restriction from the original expression.

$$\frac{x+1}{x-2}+\frac{2}{x-2}$$

Reference answer

(x+3)/(x-2); x≠2.

$$\frac{x+3}{x-2}$$

1. First record x≠2 from the original denominators and any divisor.
2. The denominators already match; add the numerators only.
3. (x+1)/(x-2)+(2)/(x-2) = (x+1+2)/(x-2).
4. Therefore (x+3)/(x-2), with x≠2 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Independent practice 5

Simplify and retain every restriction from the original expression.

$$\frac{x^2-9}{x-1}\times\frac{x-1}{x+3}$$

Reference answer

x-3; x≠1, -3.

$$x-3$$

1. First record x≠1, -3 from the original denominators and any divisor.
2. Factor first, then cancel common factors; never cancel separate added terms.
3. (x^2-9)/(x-1)×(x-1)/(x+3) = ((x-3)(x+3)(x-1))/((x-1)(x+3)).
4. Therefore x-3, with x≠1, -3 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Independent practice 6

Simplify and retain every restriction from the original expression.

$$\frac{2}{x}-\frac{1}{x+1}$$

Reference answer

(x+2)/(x(x+1)); x≠0, -1.

$$\frac{x+2}{x(x+1)}$$

1. First record x≠0, -1 from the original denominators and any divisor.
2. Use a common denominator and multiply the entire numerator by the required factor.
3. (2)/(x)-(1)/(x+1) = (2(x+1)-x)/(x(x+1)).
4. Therefore (x+2)/(x(x+1)), with x≠0, -1 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Independent practice 7

Evaluate $(-2)^{-2}$. Show the reciprocal before calculating.

$$(-2)^{-2}$$

Reference answer

1/4.

$$\frac{1}{4}$$

1. The whole negative base is squared after taking the reciprocal.
2. $(-2)^{-2}=1/(-2)^{2}=1/4$.

- Use the stated relationship and show the calculation.
- State the result with its unit where appropriate.

Independent practice 8

Write 0.00056 in scientific notation with $1 \leq |a| < 10$.

$$0.00056$$

Reference answer

5.6×10^{-4}.

$$5.6 \times 10^{-4}$$

1. Move the decimal point to obtain mantissa 5.6, whose absolute value is between 1 and 10.
2. Compensate with 10^{-4}, so $0.00056=5.6 \times 10^{-4}$.
3. Check the sign and the decimal order of magnitude.

- Use a normalized nonzero mantissa and an integer exponent.
- Preserve the original value and sign.

Exit question 9

Simplify and retain every restriction from the original expression.

$$\frac{3}{x} \div \frac{6}{x}$$

Reference answer

(1)/(2); $x \neq 0$.

$$\frac{1}{2}$$

1. First record $x \neq 0$ from the original denominators and any divisor.
2. Replace division by multiplication by the reciprocal; the original divisor must remain nonzero.
3. $(3)/(x) \div (6)/(x) = (3)/(x) \times (x)/(6)$.
4. Therefore (1)/(2), with $x \neq 0$ still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Exit question 10

Write 0.000072 in scientific notation with 1≤|a|<10.

$$0.000072$$

Reference answer

7.2×10^{-5}.

$$7.2 \times 10^{-5}$$

1. Move the decimal point to obtain mantissa 7.2, whose absolute value is between 1 and 10.
2. Compensate with 10^{-5}, so $0.000072=7.2\times10^{-5}$.
3. Check the sign and the decimal order of magnitude.

- Use a normalized nonzero mantissa and an integer exponent.
- Preserve the original value and sign.

Further practice 11

Simplify and retain every restriction from the original expression.

$$(\frac{1}{x}-\frac{1}{x+1})(x^2+x)$$

Reference answer

1; x≠0, -1.

$$1$$

1. First record x≠0, -1 from the original denominators and any divisor.
2. Complete the subtraction first, factor $x^2+x=x(x+1)$, then multiply and cancel.
3. $((1)/(x)-(1)/(x+1))(x^2+x) = (1)/(x(x+1))x(x+1)$.
4. Therefore 1, with x≠0, -1 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Further practice 12

Simplify and retain every restriction from the original expression.

$$\frac{x+2}{x}-\frac{2}{x}$$

Reference answer

1; x≠0.

$$1$$

1. First record x≠0 from the original denominators and any divisor.
2. Subtract the entire second numerator: (x+2)−2=x.
3. $(x+2)/(x)-(2)/(x) = (x+2-2)/(x)$.
4. Therefore 1, with x≠0 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Further practice 13

Write -0.00032 in scientific notation with 1≤|a|<10.

$$-0.00032$$

Reference answer

-3.2×10⁻⁴.

$$-3.2 \times 10^{-4}$$

1. Move the decimal point to obtain mantissa -3.2, whose absolute value is between 1 and 10.
2. Compensate with 10^{-4}, so -0.00032=-3.2×10⁻⁴.
3. Check the sign and the decimal order of magnitude.

- Use a normalized nonzero mantissa and an integer exponent.
- Preserve the original value and sign.

Further practice 14

Evaluate $(-2/3)^{-2}$.

$$(-2/3)^{-2}$$

Reference answer

9/4.

$$\frac{9}{4}$$

1. Invert the base and change the exponent to positive 2.
2. $(-3/2)^2=9/4$.

- Use the stated relationship and show the calculation.
- State the result with its unit where appropriate.

Extension 15

Simplify and retain every restriction from the original expression.

$$(\frac{1}{x-1}-\frac{1}{x+1})\div\frac{2}{x^2-1}$$

Reference answer

1; x≠-1, 1.

$$1$$

1. First record x≠-1, 1 from the original denominators and any divisor.
2. The bracket simplifies to $2/(x^2-1)$, then divides by that same nonzero expression.
3. $((1)/(x-1)-(1)/(x+1))\div(2)/(x^2-1)=(2)/(x^2-1)\div(2)/(x^2-1)$.
4. Therefore 1, with x≠-1, 1 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Extension 16

Calculate 0.0003×0.0002 and express the product in scientific notation.

$$0.0003 \times 0.0002$$

Reference answer

6×10⁻⁸.

$$6 \times 10^{-8}$$

1. $0.0003=3\times10^{-4}$ and $0.0002=2\times10^{-4}$.
2. Multiply the mantissas and add the exponents:$3\times2\times10^{-8}=6\times10^{-8}$.
3. The mantissa 6 is normalized; the product is 0.00000006.

- Use a normalized nonzero mantissa and an integer exponent.
- Preserve the original value and sign.