Rational expressions: meaning, domain and cancellation — Teacher answers

Year 8, upper volume • text pp. 79–82 • foundation lessons 40–41

Use the worked model and guided attempt selectively, then the eight core questions. Keep further practice and extensions optional. Ask for a reason and an original-equation or expansion check.

Worked example 1

Simplify and retain every restriction from the original expression.

$$\frac{x^2 - 9}{x - 3}$$

Reference answer

x+3; x≠3.

$$x + 3$$

1. First record x≠3 from the original denominators and any divisor.
2. Factor first, then cancel common factors; never cancel separate added terms.
3. $(x^2-9)/(x-3) = ((x-3)(x+3))/(x-3)$.
4. Therefore x+3, with x≠3 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Guided attempt 2

Simplify and retain every restriction from the original expression.

$$\frac{2x^2 + 4x}{2x}$$

1. Original restriction: x≠___.
2. Factor the numerator as 2x(___); cancel the factor ___, not added terms.

Reference answer

x+2; x≠0.

$$x + 2$$

1. First record x≠0 from the original denominators and any divisor.
2. Factor first, then cancel common factors; never cancel separate added terms.
3. $(2x^2+4x)/(2x) = (2x(x+2))/(2x)$.
4. Therefore x+2, with x≠0 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Prerequisite 3

Which displayed expression has a variable in its denominator?

$$\frac{1}{x + 2}$$

A. x^2 B. x+1
C. 1/(x+2) D. x/3

Reference answer

1/(x+2)

$$\frac{1}{x + 2}$$

1. Inspect the denominator of the original expression.
2. Only 1/(x+2) has a variable denominator; it excludes x=−2.

- Choose the statement justified by the stated conditions and explain why.

Prerequisite 4

Evaluate 1/(x−2) at x=3. Check the denominator first.

$$1/(3-2)$$

Reference answer

1.

1. The denominator is 3−2=1, which is nonzero.
2. The value is 1/1=1.

- Use the stated relationship and show the calculation.
- State the result with its unit where appropriate.

Independent practice 5

State the domain restriction. Does the fraction simplify by cancellation?

$$\frac{x+1}{x-4}$$

Reference answer

(x+1)/(x-4); x≠4.

$$\frac{x+1}{x-4}$$

1. First record x≠4 from the original denominators and any divisor.
2. No common factor cancels; solve x−4=0 to find the excluded value.
3. (x+1)/(x-4) = (x+1)/(x-4).
4. Therefore (x+1)/(x-4), with x≠4 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Independent practice 6

For which x is (x−5)/(x+1) equal to zero? Check that the expression is defined.

$$\frac{x-5}{x+1}=0$$

Reference answer

x=5.

1. A fraction is zero when the numerator is zero and the denominator is nonzero.
2. x−5=0 gives x=5. At x=5, the denominator is 6≠0.
3. The original fraction is 0/6=0, so x=5 is allowed.

- Show a complete equation after clearing denominators.
- Check original denominators and the original equality.

Independent practice 7

Simplify and retain every restriction from the original expression.

$$\frac{x^2-16}{x+4}$$

Reference answer

x-4; x≠-4.

$$x-4$$

1. First record x≠-4 from the original denominators and any divisor.
2. Factor first, then cancel common factors; never cancel separate added terms.
3. (x^2-16)/(x+4) = ((x-4)(x+4))/(x+4).
4. Therefore x-4, with x≠-4 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Independent practice 8

Simplify and retain every restriction from the original expression.

$$\frac{x^2 + 3x}{x}$$

Reference answer

x+3; x≠0.

$$x + 3$$

1. First record x≠0 from the original denominators and any divisor.
2. Factor first, then cancel common factors; never cancel separate added terms.
3. (x^2+3x)/(x) = (x(x+3))/(x).
4. Therefore x+3, with x≠0 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Exit question 9

Simplify (x−2)/(x−2). Does the simplified value permit x=2? State the original restriction.

$$\frac{x - 2}{x - 2}$$

Reference answer

1; x≠2.

$$1$$

1. First record x≠2 from the original denominators and any divisor.
2. The common factor cancels only when it is nonzero; at x=2 the original expression is 0/0.
3. (x-2)/(x-2) = 1.
4. Therefore 1, with x≠2 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Exit question 10

Simplify and retain every restriction from the original expression.

$$\frac{x + 2}{x}$$

Reference answer

1+(2)/(x); x≠0.

$$1 + \frac{2}{x}$$

1. First record x≠0 from the original denominators and any divisor.
2. Split the division over numerator terms; x is not a factor of the entire numerator x+2.
3. (x+2)/(x) = (x)/(x)+(2)/(x).
4. Therefore 1+(2)/(x), with x≠0 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Further practice 11

Simplify and retain every restriction from the original expression.

$$\frac{3x}{6x}$$

Reference answer

(1)/(2); x≠0.

$$\frac{1}{2}$$

1. First record x≠0 from the original denominators and any divisor.
2. Factor first, then cancel common factors; never cancel separate added terms.
3. (3x)/(6x) = (3)/(6).
4. Therefore (1)/(2), with x≠0 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Further practice 12

Simplify and retain every restriction from the original expression.

$$\frac{x^2 - 1}{x - 1}$$

Reference answer

x+1; x≠1.

$$x + 1$$

1. First record x≠1 from the original denominators and any divisor.
2. Factor first, then cancel common factors; never cancel separate added terms.
3. (x^2-1)/(x-1) = ((x-1)(x+1))/(x-1).
4. Therefore x+1, with x≠1 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Further practice 13

Simplify and retain every restriction from the original expression.

$$\frac{x^2 + 2x}{x + 2}$$

Reference answer

x; x≠-2.

$$x$$

1. First record x≠-2 from the original denominators and any divisor.
2. Factor first, then cancel common factors; never cancel separate added terms.
3. (x^2+2x)/(x+2) = (x(x+2))/(x+2).
4. Therefore x, with x≠-2 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Further practice 14

Simplify and retain every restriction from the original expression.

$$\frac{x-1}{x+1}$$

Reference answer

(x-1)/(x+1); x≠-1.

$$\frac{x-1}{x+1}$$

1. First record x≠-1 from the original denominators and any divisor.
2. The numerator and denominator have no common nonconstant factor. Keep x≠−1.
3. (x-1)/(x+1) = (x-1)/(x+1).
4. Therefore (x-1)/(x+1), with x≠-1 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Extension 15

After cancellation the formula is x+2. Explain why it is not the same original expression on ALL real inputs.

$$\frac{x^2-4}{x-2}$$

Reference answer

They agree for x≠2. The original is undefined at 2, while the unrestricted polynomial has value 4.

$$x+2$$

1. First record x≠2 from the original denominators and any divisor.
2. Factor first, then cancel common factors; never cancel separate added terms.
3. (x^2-4)/(x-2) = ((x-2)(x+2))/(x-2).
4. Therefore x+2, with x≠2 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Extension 16

Simplify, and explain why x=1 remains forbidden even though the final formula is just x.

$$\frac{x^2-x}{x-1}$$

Reference answer

x; x≠1.

$$x$$

1. First record x≠1 from the original denominators and any divisor.
2. Factor first, then cancel common factors; never cancel separate added terms.
3. (x^2-x)/(x-1) = (x(x-1))/(x-1).
4. Therefore x, with x≠1 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.