Rational equations: checking and applications — Teacher answers

Year 8, upper volume • text pp. 97–104 • foundation lessons 49–52

Use the worked model and guided attempt selectively, then the eight core questions. Keep further practice and extensions optional. Ask for a reason and an original-equation or expansion check.

Worked example 1

Solve the equation. State the restrictions and check every candidate in the original equation.

$$\frac{x}{x-2} = \frac{2}{x-2}$$

Reference answer

No solution.

1. The original equation requires x≠2.
2. Multiplying both sides by x −2 gives x=2.
3. That candidate makes both original denominators zero, so it is rejected.
4. The original equation therefore has no solution.

- Show a complete equation after clearing denominators.
- Check original denominators and the original equality.

Guided attempt 2

Solve the equation. State the restrictions and check every candidate in the original equation.

$$\frac{3}{x-1} = 2$$

1. Restriction x≠___; cleared equation 3=___.
2. Candidate x=___; original denominator=___, so ___.

Reference answer

x=5/2.

1. The restriction is x≠1.
2. Multiply every term by x −1:3=2(x−1).
3. 3=2x−2, so x=5/2.
4. Check:3/(5/2−1)=3/(3/2)=2, and 5/2≠1.

- Show a complete equation after clearing denominators.
- Check original denominators and the original equality.

Prerequisite 3

Before solving any equation containing 1/(x−3), state its forbidden value.

$$\frac{1}{x-3}$$

Reference answer

(1)/(x-3); x≠3.

$$\frac{1}{x-3}$$

1. First record x≠3 from the original denominators and any divisor.
2. The denominator x−3 must be nonzero.
3. (1)/(x-3) = (1)/(x-3).
4. Therefore (1)/(x-3), with x≠3 still required.

- Use factor cancellation or a correct common denominator.
- Retain all original restrictions, including a nonzero divisor.

Prerequisite 4

A team completes one job in 6 days at a constant rate. What fraction of the job does it complete per day?

Reference answer

1/6.

$$\frac{1}{6}$$

1. Treat the whole job as 1.
2. Daily rate=whole job÷days=1/6.

- Use the stated relationship and show the calculation.
- State the result with its unit where appropriate.

Independent practice 5

Solve the equation. State the restrictions and check every candidate in the original equation.

$$\frac{2}{x-1}=1$$

Reference answer

x=3.

1. Record x≠1.
2. 2=x−1 gives x=3.
3. Check 2/(3−1)=1 and the denominator 2 is nonzero.

- Show a complete equation after clearing denominators.
- Check original denominators and the original equality.

Independent practice 6

Solve the equation. State the restrictions and check every candidate in the original equation.

$$\frac{x+1}{x-2}=\frac{3}{x-2}$$

Reference answer

No solution.

1. The original restriction is x≠2.
2. Clearing denominators gives x+1=3, so the candidate is x=2.
3. The candidate is forbidden; the original equation has no solution.

- Show a complete equation after clearing denominators.
- Check original denominators and the original equality.

Independent practice 7

Team A alone takes 12 days. A and B together take 4 days. Let x be B’ s days alone. Form and solve a rate equation.

Reference answer

x=6 days; it is positive and satisfies the original model.

$$\frac{1}{12}+\frac{1}{x}=\frac{1}{4}$$

1. A’ s rate is 1/12 and B’ s rate is 1/x; require x>0.
2. The rates add:1/12+1/x=1/4.
3. Thus 1/x=1/6, so x=6 days.
4. Check 1/12+1/6=1/4;6 is positive and nonzero.

- Show a complete equation after clearing denominators.
- Check original denominators and the original equality.

Independent practice 8

Two travellers take the same time. One covers 90 km at x km/h; the other covers 120 km at (x+10) km/h. Find x.

Reference answer

x=30 km/h; it is positive and satisfies the original model.

$$\frac{90}{x} = \frac{120}{x+10}$$

1. Times are 90/x and 120/(x+10); require x>0.
2. Equal times give 90(x+10)=120x after clearing denominators.
3. 900=30x, so x=30 km/h.
4. Check 90/30=120/40=3 hours; both speeds are positive.

- Show a complete equation after clearing denominators.
- Check original denominators and the original equality.

Exit question 9

Solve the equation. State the restrictions and check every candidate in the original equation.

$$\frac{1}{x-1} = \frac{2}{x-1}$$

Reference answer

No solution.

1. The original domain requires x≠1.
2. Clearing the common denominator gives 1=2, which is impossible.
3. Therefore there is no solution; x=1 cannot be used to evade the contradiction.

- Show a complete equation after clearing denominators.
- Check original denominators and the original equality.

Exit question 10

Solve the equation. State the restrictions and check every candidate in the original equation.

$$\frac{4}{x} = 2$$

Reference answer

x=2.

1. Record x≠0, then 4=2x.
2. x=2; check 4/2=2 in the original equation.

- Show a complete equation after clearing denominators.
- Check original denominators and the original equality.

Further practice 11

Solve the equation. State the restrictions and check every candidate in the original equation.

$$\frac{3}{x^2-4} = \frac{1}{x-2}$$

Reference answer

x=1.

1. Factor $x^2-4=(x-2)(x+2)$, so x≠−2,2.
2. Multiply by the full denominator:3=x+2.
3. x=1; check 3/(1−4)=−1=1/(1−2).

- Show a complete equation after clearing denominators.
- Check original denominators and the original equality.

Further practice 12

Solve the equation. State the restrictions and check every candidate in the original equation.

$$\frac{1}{2x}=\frac{2}{x+3}$$

Reference answer

x=1.

1. The restrictions are x≠0,−3.
2. Multiply by 2x(x+3):x+3=4x, so x=1.
3. Check 1/(2×1)=2/(1+3)=1/2.

- Show a complete equation after clearing denominators.
- Check original denominators and the original equality.

Further practice 13

A alone takes 6 days and B alone takes 3 days. Let x be their days working together. Form and solve a work equation.

Reference answer

x=2 days; it is positive and satisfies the original model.

$$\frac{x}{6}+\frac{x}{3}=1$$

1. In x days the work fractions are x/6 and x/3.
2. x/6+x/3=1 gives x+2x=6, so x=2.
3. Check 2/6+2/3=1;2 days is positive.

- Show a complete equation after clearing denominators.
- Check original denominators and the original equality.

Further practice 14

Two trips take equal time:180 km at x km/h and 240 km at (x+20) km/h. Find x.

Reference answer

x=60 km/h; it is positive and satisfies the original model.

$$\frac{180}{x}=\frac{240}{x+20}$$

1. Use time=distance/speed and require x>0.
2. 180(x+20)=240x gives 3600=60x, so x=60.
3. Check 180/60=240/80=3 hours.

- Show a complete equation after clearing denominators.
- Check original denominators and the original equality.

Extension 15

A alone takes 8 days, and A+B together take 3 days. How many days does B take alone? Let this number be x.

Reference answer

x=24/5 days; it is positive and satisfies the original model.

$$\frac{1}{8}+\frac{1}{x}=\frac{1}{3}$$

1. Require x>0 and add the rates:1/8+1/x=1/3.
2. 1/x=1/3−1/8=5/24, so x=24/5 days.
3. Check 1/8+5/24=3/24+5/24=1/3; the noninteger time is valid.

- Show a complete equation after clearing denominators.
- Check original denominators and the original equality.

Extension 16

A cyclist travels 12 km at x km/h. A bus travels the same route at 2x km/h but starts 1/2 hour later; they arrive together. Find x.

Reference answer

x=12 km/h; it is positive and satisfies the original model.

$$\frac{12}{x} = \frac{12}{2x} + \frac{1}{2}$$

1. The cyclist’ s time exceeds the bus time by 1/2 hour; require x>0.
2. 12/x=6/x+1/2 gives 6/x=1/2, so x=12.
3. Check cyclist 1 hour, bus 1/2 hour, and the delayed starts yield the same arrival time.

- Show a complete equation after clearing denominators.
- Check original denominators and the original equality.