Square roots: meaning, simplification and domain — Teacher answers

Year 8, lower volume • text pp. 1–4 • textbook lessons 1–2

Use the two prerequisite checks to choose support. Project the worked and guided tasks, then collect the eight paper core questions. Follow-ups and extensions are optional; accept other complete valid methods. This page does not save learners' paper answers.

Ordinary route: prerequisites 4, short inquiry 5, worked/guided 8, independent 15, correction 5, exit 3 minutes. Advanced route: inquiry 18, organise 4, selected practice 13, exit 5 minutes.

Prerequisites: Squares, nonnegative numbers and linear inequalities.

Ordinary route: use the first fixed case for the five-minute demonstration. Advanced route: complete the full investigation.

1. 3′ Predict the values of $\sqrt{x^2}$ for x=3, −3 and 0. Does the original sign survive? Write a reason before calculating.
2. 8′ On paper, make columns for x, its square and the principal root of that square. Then test the expression $\sqrt{}$(x−3) at x=1,3,7; mark which inputs are allowed over the reals.
3. 5′ Explain $\sqrt{x^2} = |x|$ and the condition $x - 3 \geq 0$. A few examples suggest the rule; justify it using the definition of the nonnegative root.
4. 2′ Individually simplify $\sqrt{(-5)^2}$ and distinguish it from solving $x^2 = 25$.

Worked example 1

Simplify exactly. State any restriction that is needed.

$$\sqrt{(-7)^2}$$

Reference answer

7

$$7$$

1. The square-root symbol gives the nonnegative root; a real radicand must be nonnegative.
2. An intermediate form is $\sqrt{49} = 7$.
3. Check the original expression and the sign of the square-root value.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Guided attempt 2

Simplify exactly. State any restriction that is needed.

$$\sqrt{50}$$

1. Complete the square-factor split: $50 = 25 \times \square$.
2. Then $\sqrt{50} = \sqrt{25}\sqrt{\square} = \square\sqrt{2}$.
3. Check that the coefficient is nonnegative and the square of the result is 50.

Reference answer

$5\sqrt{2}$

$$5\sqrt{2}$$

1. Extract square factors first. The square-root symbol denotes the nonnegative root.
2. An intermediate form is $\sqrt{25 \times 2} = 5\sqrt{2}$.
3. Check the original expression and the sign of the square-root value.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Prerequisite 3

Simplify exactly. State any restriction that is needed.

$$\sqrt{49}$$

Reference answer

7

$$7$$

1. The square-root symbol gives the nonnegative root; a real radicand must be nonnegative.
2. The simplified form is 7.
3. Check the original expression and the sign of the square-root value.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Prerequisite 4

Is this expression defined over the real numbers? Explain.

$$\sqrt{-4}$$

Reference answer

No real value; the constant radicand −4 is negative.

$$\varnothing$$

1. Every radicand must be nonnegative.
2. Solve the resulting linear inequalities, then take their common part.
3. The complete permitted set is $\varnothing$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 5

Find all real values of x for which the expression is defined.

$$\sqrt{x-3}$$

Reference answer

$x \geq 3$

$$x \geq 3$$

1. Every radicand must be nonnegative.
2. Solve the resulting linear inequalities, then take their common part.
3. The complete permitted set is $x \geq 3$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 6

Find all real values of x for which the expression is defined.

$$\sqrt{5-2x}$$

Reference answer

$x \leq \frac{5}{2}$

$$x \leq \frac{5}{2}$$

1. Every radicand must be nonnegative.
2. Solve the resulting linear inequalities, then take their common part.
3. The complete permitted set is $x \leq \frac{5}{2}$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 7

Simplify exactly. State any restriction that is needed.

$$\sqrt{x^2}$$

Reference answer

$|x|$

$$|x|$$

1. A square root cannot be negative, even when x is negative.
2. The simplified form is $|x|$.
3. Check the original expression and the sign of the square-root value.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 8

Find all real values of x for which the expression is defined.

$$\frac{1}{\sqrt{x+2}}$$

Reference answer

$x > -2$

$$x > -2$$

1. Every radicand must be nonnegative; a square root in a denominator must be strictly positive.
2. Solve the resulting linear inequalities, then take their common part.
3. The complete permitted set is $x > -2$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Exit question 9

Simplify exactly. State any restriction that is needed.

$$\sqrt{0}$$

Reference answer

0

$$0$$

1. The square-root symbol gives the nonnegative root; a real radicand must be nonnegative.
2. The simplified form is 0.
3. Check the original expression and the sign of the square-root value.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Exit question 10

A learner writes $\sqrt{25} = \pm 5$. Give the correct value and explain the error.

$$\sqrt{25}$$

Reference answer

5

$$5$$

1. The equation $x^2=25$ has two solutions; the square-root symbol itself denotes only 5.
2. The simplified form is 5.
3. Check the original expression and the sign of the square-root value.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 11

Simplify exactly. State any restriction that is needed.

$$\sqrt{\frac{1}{4}}$$

Reference answer

$\frac{1}{2}$

$$\frac{1}{2}$$

1. The square-root symbol gives the nonnegative root; a real radicand must be nonnegative.
2. The simplified form is $\frac{1}{2}$.
3. Check the original expression and the sign of the square-root value.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 12

Find all real values of x for which the expression is defined.

$$\sqrt{x-1} + \sqrt{4-x}$$

Reference answer

$1 \le x \le 4$

$$1 \le x \le 4$$

1. Every radicand must be nonnegative.
2. Solve the resulting linear inequalities, then take their common part.
3. The complete permitted set is $1 \le x \le 4$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 13

Simplify exactly. State any restriction that is needed.

$$\sqrt{(0-2)^2}$$

Reference answer

2

$$2$$

1. The square-root symbol gives the nonnegative root; a real radicand must be nonnegative.
2. An intermediate form is $|0-2|=2$.
3. Check the original expression and the sign of the square-root value.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 14

Simplify exactly. State any restriction that is needed.

$$(\sqrt{11})^2$$

Reference answer

11

$$11$$

1. Squaring the nonnegative square root restores its nonnegative radicand.
2. The simplified form is 11.
3. Check the original expression and the sign of the square-root value.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Extension 15

Find all real values of x for which the expression is defined.

$$\frac{\sqrt{2x+1}}{\sqrt{3-x}}$$

Reference answer

$-\frac{1}{2} \le x < 3$

$$-\frac{1}{2} \le x < 3$$

1. Every radicand must be nonnegative; a square root in a denominator must be strictly positive.
2. Solve the resulting linear inequalities, then take their common part.
3. The complete permitted set is $-\frac{1}{2} \le x < 3$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Extension 16

Use this value to disprove the claim $\sqrt{x^2} = x$ for every real x.

$$\sqrt{(-2)^2}$$

Reference answer

2

$$2$$

1. At x=−2, the root is 2 while x is −2. The correct identity uses absolute value.
2. The simplified form is 2.
3. Check the original expression and the sign of the square-root value.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.