Pythagoras converse: conditions and combined problems — Teacher answers

Year 8, lower volume • text pp. 19–26 • textbook lessons 10–13

Use the two prerequisite checks to choose support. Project the worked and guided tasks, then collect the eight paper core questions. Follow-ups and extensions are optional; accept other complete valid methods. This page does not save learners' paper answers.

Ordinary route: prerequisites 4, short inquiry 5, worked/guided 8, independent 15, correction 5, exit 3 minutes. Advanced route: inquiry 18, organise 4, selected practice 13, exit 5 minutes.

Prerequisites: Triangle inequalities, squares and the Pythagorean theorem.

Ordinary route: use the first fixed case for the five-minute demonstration. Advanced route: complete the full investigation.

1. 3′ Predict which length cards make right triangles: (5,12,13), (5,12,14), (2,3,5). Do not rely on appearance.
2. 8′ On paper, first check positive lengths and the strict triangle inequality. For each valid triangle identify its longest side, then compare its square with the sum of the other two squares. The existing right-triangle engine cannot test three arbitrary sides for this converse.
3. 5′ Explain why the first is right, the second is not right, and the third is degenerate. Distinguish triangle existence from the square equality.
4. 2′ Individually test (8,15,17), and name the angle opposite the longest side in the conclusion.

Worked example 1

Can sides 13, 5, 12 cm form a right triangle? Check existence and identify the longest side first.

Reference answer

Yes: the positive lengths satisfy the triangle inequalities and the Pythagorean converse.

1. The longest side is 13; the shorter sum is 17.
2. $5^2 + 12^2 = 169$ and $13^2 = 169$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Guided attempt 2

Can sides 8, 15, 17 cm form a right triangle? Check existence and identify the longest side first.

1. Existence and longest side: $8 + 15 > \square$; longest side = ____.
2. $8^2 + 15^2 = \square$; $17^2 = \square$.
3. The two square values are ____; therefore, by ____, the angle opposite 17 cm is ____.

Reference answer

Yes: the positive lengths satisfy the triangle inequalities and the Pythagorean converse.

1. The longest side is 17; the shorter sum is 23.
2. $8^2 + 15^2 = 289$ and $17^2 = 289$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Prerequisite 3

Can sides 3, 4, 5 cm form a right triangle? Check existence and identify the longest side first.

Reference answer

Yes: the positive lengths satisfy the triangle inequalities and the Pythagorean converse.

1. The longest side is 5; the shorter sum is 7.
2. $3^2 + 4^2 = 25$ and $5^2 = 25$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Prerequisite 4

Can sides 2, 3, 5 cm form a right triangle? Check existence and identify the longest side first.

Reference answer

No: the required squared equality fails (or the lengths do not form a triangle).

1. The longest side is 5; the shorter sum is 5.
2. $2^2 + 3^2 = 13$ and $5^2 = 25$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 5

Can sides 5, 12, 14 cm form a right triangle? Check existence and identify the longest side first.

Reference answer

No: the required squared equality fails (or the lengths do not form a triangle).

1. The longest side is 14; the shorter sum is 17.
2. $5^2 + 12^2 = 169$ and $14^2 = 196$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 6

Can sides 6, 8, 10 cm form a right triangle? Check existence and identify the longest side first.

Reference answer

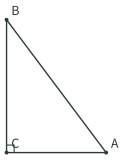
Yes: the positive lengths satisfy the triangle inequalities and the Pythagorean converse.

1. The longest side is 10; the shorter sum is 14.
2. $6^2 + 8^2 = 100$ and $10^2 = 100$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 7

A rectangular panel measures 9 cm by 12 cm. A straight brace joins opposite corners. Find the brace length.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

15 cm.

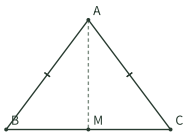
$$15$$

1. The right angle is given; add the two leg squares to find the hypotenuse square.
2. The required square is $9^2 + 12^2$; take the positive square root.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 8

AB=AC=5 cm and BC=6 cm. M is the midpoint of BC. Find AM and the triangle area, stating why AM is perpendicular to BC.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

AM=4 cm; area=12 cm^2. In an isosceles triangle, the median from the apex is also an altitude.

1. BM=3 cm; the apex median is perpendicular to BC.
2. AM^2=25−9=16, so AM=4. Area=$\frac{1}{2} \times 6 \times 4 = 12$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Exit question 9

Can sides 4, 5, 6 cm form a right triangle? Check existence and identify the longest side first.

Reference answer

No: the required squared equality fails (or the lengths do not form a triangle).

1. The longest side is 6; the shorter sum is 9.
2. $4^2 + 5^2 = 41$ and $6^2 = 36$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Exit question 10

Can sides 25, 7, 24 cm form a right triangle? Check existence and identify the longest side first.

Reference answer

Yes: the positive lengths satisfy the triangle inequalities and the Pythagorean converse.

1. The longest side is 25; the shorter sum is 31.
2. $7^2 + 24^2 = 625$ and $25^2 = 625$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 11

Can sides 1.5, 2, 2.5 cm form a right triangle? Check existence and identify the longest side first.

Reference answer

Yes: the positive lengths satisfy the triangle inequalities and the Pythagorean converse.

1. The longest side is 2.5; the shorter sum is 3.5.
2. $1.5^2 + 2^2 = 6.25$ and $2.5^2 = 6.25$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 12

Can sides 5, 5, 6 cm form a right triangle? Check existence and identify the longest side first.

Reference answer

No: the required squared equality fails (or the lengths do not form a triangle).

1. The longest side is 6; the shorter sum is 10.
2. $5^2 + 5^2 = 50$ and $6^2 = 36$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 13

A rectangle has diagonal 5 cm and width 3 cm. Find its length.

Reference answer

4 cm.

$$4$$

1. The hypotenuse is known; subtract the square of the known leg from its square.
2. The required square is $5^2 - 3^2$; take the positive square root.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 14

Can sides 29, 20, 21 cm form a right triangle? Check existence and identify the longest side first.

Reference answer

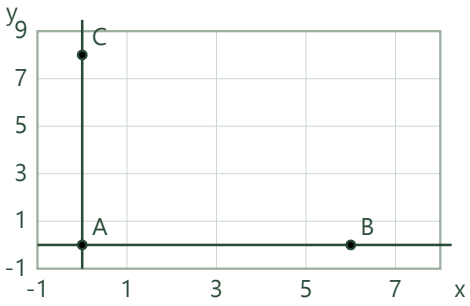
Yes: the positive lengths satisfy the triangle inequalities and the Pythagorean converse.

1. The longest side is 29; the shorter sum is 41.
2. $20^2 + 21^2 = 841$ and $29^2 = 841$.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Extension 15

A=$(0, 0)$, B=$(6, 0)$, C=$(0, 8)$. Verify the three lengths and use the converse to identify the right angle.



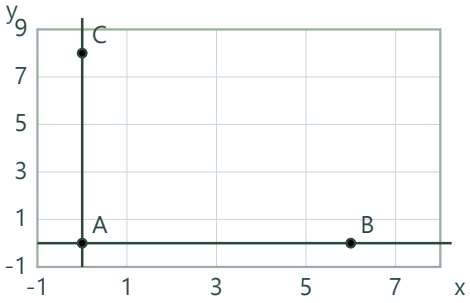
Use the stated conditions and the labelled scales.

Reference answer

AB=6, AC=8, BC=10; ∠BAC is 90°.

1. Horizontal and vertical distances give AB=6 and AC=8. The diagonal length is 10.
2. $6^2+8^2=10^2$, so the angle opposite BC is a right angle at A.

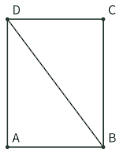
- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.



Use the stated conditions and the labelled scales.

Extension 16

A parallelogram has adjacent sides 6 cm and 8 cm, and the diagonal joining their free endpoints is 10 cm. Prove it is a rectangle.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

The included angle is 90° by the Pythagorean converse. A parallelogram with one right angle is a rectangle.

1. The sides 6,8,10 form a triangle and satisfy $6^2+8^2=10^2$.
2. The angle between the two given adjacent sides is therefore right; use the parallelogram premise for the rectangle criterion.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.