Special products and polynomial division — Teacher answers

Year 8, upper volume • text pp. 63–72 • foundation lessons 32–36

Use the worked model and guided attempt selectively, then the eight core questions. Keep further practice and extensions optional. Ask for a reason and an original-equation or expansion check.

Worked example 1

Simplify completely, showing the relevant rule.

$$(2x-3)^2$$

Reference answer

$4x^2-12x+9$.

$$4x^2-12x+9$$

1. Use $(a\pm b)^2=a^2\pm 2ab+b^2$; retain the middle term.
2. $(2x-3)^2=(2x)^2-2\cdot(2x)\cdot 3+3^2$.
3. Collect like terms to obtain $4x^2-12x+9$.
4. Check the signs, coefficients and exponents in every term.

- Show the rule and a complete intermediate line.
- Give an algebraically equivalent simplified expression.

Guided attempt 2

Simplify completely, showing the relevant rule.

$$(3x+2)(3x-2)$$

1. The matching terms are a=___ and b=___.
2. a^2-b^2=___−___=___.

Reference answer

$9x^2-4$.

$$9x^2-4$$

1. Use $(a+b)(a-b)=a^2-b^2$, identifying both terms first.
2. $(3x+2)(3x-2)=(3x)^2-2^2$.
3. Collect like terms to obtain $9x^2-4$.
4. Check the signs, coefficients and exponents in every term.

- Show the rule and a complete intermediate line.
- Give an algebraically equivalent simplified expression.

Prerequisite 3

Evaluate $(-3)^2$, including the sign.

$$(-3)^2$$

Reference answer

9.

1. The negative sign is inside the brackets, so both factors are −3.
2. (−3)(−3)=9.

- Use the stated relationship and show the calculation.
- State the result with its unit where appropriate.

Prerequisite 4

Simplify; retain the original divisor restriction $x \neq 0$.

$$\frac{x^5}{x^2}$$

Reference answer

x^3; $x \neq 0$.

$$x^3$$

1. Divide coefficients and subtract same-base exponents; divide every polynomial term. The divisor must be nonzero.
2. $\frac{x^5}{x^2} = x^{5-2}$.
3. Collect like terms to obtain x^3.
4. Check the signs, coefficients and exponents in every term.

- Show the rule and a complete intermediate line.
- Give an algebraically equivalent simplified expression.

Independent practice 5

Simplify completely, showing the relevant rule.

$$(x+5)(x-5)$$

Reference answer

x^2-25.

$$x^2 - 25$$

1. Use $(a+b)(a-b)=a^2-b^2$, identifying both terms first.
2. $(x+5)(x-5) = x^2 - 5^2$.
3. Collect like terms to obtain x^2-25.
4. Check the signs, coefficients and exponents in every term.

- Show the rule and a complete intermediate line.
- Give an algebraically equivalent simplified expression.

Independent practice 6

Simplify completely, showing the relevant rule.

$$(x+4)^2$$

Reference answer

$x^2+8x+16$.

$$x^2 + 8x + 16$$

1. Use $(a \pm b)^2=a^2 \pm 2ab+b^2$; retain the middle term.
2. $(x+4)^2 = x^2 + 2 \cdot x \cdot 4 + 4^2$.
3. Collect like terms to obtain $x^2+8x+16$.
4. Check the signs, coefficients and exponents in every term.

- Show the rule and a complete intermediate line.
- Give an algebraically equivalent simplified expression.

Independent practice 7

Simplify; retain the original divisor restriction $x \neq 0$.

$$\frac{x^3}{x^3}$$

Reference answer

1; $x \neq 0$.

$$1$$

1. Divide coefficients and subtract same-base exponents; divide every polynomial term. The divisor must be nonzero.
2. $\frac{x^3}{x^3} = x^{3-3} = x^0$.
3. Collect like terms to obtain 1.
4. Check the signs, coefficients and exponents in every term.

- Show the rule and a complete intermediate line.
- Give an algebraically equivalent simplified expression.

Independent practice 8

Simplify; retain the original divisor restriction $x \neq 0$.

$$\frac{6x^3 - 9x^2}{3x}$$

Reference answer

$2x^2-3x$; $x \neq 0$.

$$2x^2 - 3x$$

1. Divide coefficients and subtract same-base exponents; divide every polynomial term. The divisor must be nonzero.
2. $\frac{6x^3-9x^2}{3x} = \frac{6x^3}{3x} - \frac{9x^2}{3x}$.
3. Collect like terms to obtain $2x^2-3x$.
4. Check the signs, coefficients and exponents in every term.

- Show the rule and a complete intermediate line.
- Give an algebraically equivalent simplified expression.

Exit question 9

Simplify completely, showing the relevant rule.

$$(x-2)^2$$

Reference answer

x^2-4x+4.

$$x^2 - 4x + 4$$

1. Use $(a \pm b)^2 = a^2 \pm 2ab + b^2$; retain the middle term.
2. $(x-2)^2 = x^2 - 2 \cdot x \cdot 2 + 2^2$.
3. Collect like terms to obtain x^2-4x+4.
4. Check the signs, coefficients and exponents in every term.

- Show the rule and a complete intermediate line.
- Give an algebraically equivalent simplified expression.

Exit question 10

Simplify completely, showing the relevant rule.

$$\frac{4x^2+8x}{4}$$

Reference answer

x^2+2x.

$$x^2+2x$$

1. Divide coefficients and subtract same-base exponents; divide every polynomial term. The divisor must be nonzero.
2. $\frac{4x^2+8x}{4}=\frac{4x^2}{4}+\frac{8x}{4}$.
3. Collect like terms to obtain x^2+2x.
4. Check the signs, coefficients and exponents in every term.

- Show the rule and a complete intermediate line.
- Give an algebraically equivalent simplified expression.

Further practice 11

Simplify completely, showing the relevant rule.

$$(x+6)(x-6)$$

Reference answer

x^2-36.

$$x^2-36$$

1. Use $(a+b)(a-b)=a^2-b^2$, identifying both terms first.
2. $(x+6)(x-6)=x^2-6^2$.
3. Collect like terms to obtain x^2-36.
4. Check the signs, coefficients and exponents in every term.

- Show the rule and a complete intermediate line.
- Give an algebraically equivalent simplified expression.

Further practice 12

Simplify completely, showing the relevant rule.

$$(3x+1)^2$$

Reference answer

$9x^2+6x+1$.

$$9x^2+6x+1$$

1. Use $(a\pm b)^2=a^2\pm 2ab+b^2$; retain the middle term.
2. $(3x+1)^2=(3x)^2+2\cdot(3x)\cdot 1+1^2$.
3. Collect like terms to obtain $9x^2+6x+1$.
4. Check the signs, coefficients and exponents in every term.

- Show the rule and a complete intermediate line.
- Give an algebraically equivalent simplified expression.

Further practice 13

Simplify; retain the original divisor restriction $x \neq 0$.

$$\frac{x^8}{x^5}$$

Reference answer

x^3; $x \neq 0$.

$$x^3$$

1. Divide coefficients and subtract same-base exponents; divide every polynomial term. The divisor must be nonzero.
2. $\frac{x^8}{x^5} = x^{8-5}$.
3. Collect like terms to obtain x^3.
4. Check the signs, coefficients and exponents in every term.

- Show the rule and a complete intermediate line.
- Give an algebraically equivalent simplified expression.

Further practice 14

Simplify; retain the original divisor restriction $x \neq 0$, $y \neq 0$.

$$\frac{-12x^3y^2}{3xy}$$

Reference answer

$-4x^2y$; $x \neq 0$, $y \neq 0$.

$$-4x^2y$$

1. Divide coefficients and subtract same-base exponents; divide every polynomial term. The divisor must be nonzero.
2. $\frac{-12x^3y^2}{3xy} = (-12/3)x^{3-1}y^{2-1}$.
3. Collect like terms to obtain $-4x^2y$.
4. Check the signs, coefficients and exponents in every term.

- Show the rule and a complete intermediate line.
- Give an algebraically equivalent simplified expression.

Extension 15

Simplify completely, showing the relevant rule.

$$(x+3)^2 - (x-3)^2$$

Reference answer

12x.

$$12x$$

1. Use $(a \pm b)^2 = a^2 \pm 2ab + b^2$; retain the middle term.
2. $(x+3)^2 - (x-3)^2 = (x^2+6x+9) - (x^2-6x+9)$.
3. Collect like terms to obtain 12x.
4. Check the signs, coefficients and exponents in every term.

- Show the rule and a complete intermediate line.
- Give an algebraically equivalent simplified expression.

Extension 16

Simplify completely, showing the relevant rule.

$$(2x-1)^2-(2x+1)(2x-1)$$

Reference answer

-4x+2.

$$-4x+2$$

1. Use (a+b)(a−b)=a²−b², identifying both terms first.
2. $(2x-1)^2-(2x+1)(2x-1)=(4x^2-4x+1)-(4x^2-1)$.
3. Collect like terms to obtain -4x+2.
4. Check the signs, coefficients and exponents in every term.

- Show the rule and a complete intermediate line.
- Give an algebraically equivalent simplified expression.