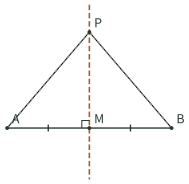
# Perpendicular bisectors: properties and the SSS converse — Teacher answers

Year 8, upper volume • text pp. 31–32 • foundation lesson 16

For the forward proof use SAS, not HL with the target hypotenuses assumed equal. In the SSS converse, omit the right-angle mark until equal adjacent angles imply 90°. Accept another justified proof.

## Worked example 1

M is the midpoint of AB. P is off AB and PM⊥AB. Prove PA=PB.




Geometry diagram; not drawn to scale except for construction tasks.

### Reference answer

△PMA≅△PMB by SAS, so PA=PB.

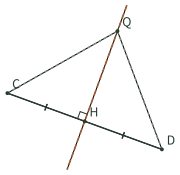
1. AM=BM because M is the midpoint of AB.
2. ∠PMA=∠PMB=90° because PM⊥AB.
3. PM=PM is common, so △PMA≅△PMB by SAS.
4. Thus PA=PB, by corresponding sides of congruent triangles.

- Use the midpoint and perpendicular conditions separately.
- State the common side and justify the congruence criterion.
- Deduce the target without assuming equal endpoint distances first.
- Accept another complete proof if every condition and deduction is justified.

## Guided attempt 2

H is the midpoint of CD, Q is off CD and QH⊥CD. Complete a proof that QC=QD.

1. CH=___ because H is ___.
2. ∠QHC=∠___=___ because ___.
3. ___ is common; the criterion is ___.
4. Therefore QC=QD by ___.




Geometry diagram; not drawn to scale except for construction tasks.

### Reference answer

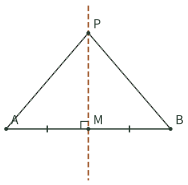
CH=DH, both included angles at H are 90°, and QH is common; SAS gives QC=QD.

1. CH=DH because H is the midpoint of CD.
2. ∠QHC=∠QHD=90° because QH⊥CD.
3. QH=QH is common, so △QHC≅△QHD by SAS.
4. Thus QC=QD, by corresponding sides of congruent triangles.

- Use the midpoint and perpendicular conditions separately.
- State the common side and justify the congruence criterion.
- Deduce the target without assuming equal endpoint distances first.
- Accept another complete proof if every condition and deduction is justified.

**Prerequisite** 3

M is the midpoint of AB and AB=10 cm. Find AM.




Geometry diagram; not drawn to scale except for construction tasks.

**Reference answer**

AM=5 cm.

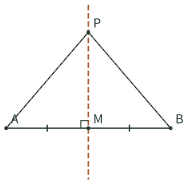
1. The midpoint divides AB into two equal segments.
2. 10÷2=5, so AM=5 cm.

- Use the correct midpoint or perpendicular-bisector property and retain cm.

**Prerequisite** 4

A,M,B are collinear, P is off AB and PM⊥AB. What is ∠PMA?

A. 45° B. 180°
C. It must equal the length AM. D. 90°




Geometry diagram; not drawn to scale except for construction tasks.

**Reference answer**

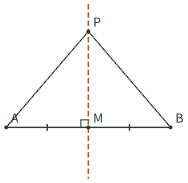
90°, by the definition of perpendicular lines.

1. MA is part of the line AB.
2. PM⊥AB therefore makes ∠PMA a right angle.

- Give an angle of 90°, not a length.

**Independent practice** 5

M is the midpoint of AB and AB=18 cm. Find AM.




Geometry diagram; not drawn to scale except for construction tasks.

**Reference answer**

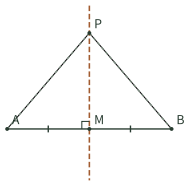
AM=9 cm.

1. The midpoint divides AB into two equal segments.
2. 18÷2=9, so AM=9 cm.

- Use the correct midpoint or perpendicular-bisector property and retain cm.

**Independent practice** 6

P lies on the perpendicular bisector of AB. If PA=8 cm, find PB and state the property.




Geometry diagram; not drawn to scale except for construction tasks.

### Reference answer

PB=8 cm: a point on the perpendicular bisector is equidistant from the endpoints.

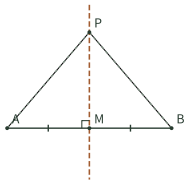
1. The point is on the perpendicular bisector, so PA=PB.
2. Substitute PA=8 cm to obtain PB=8 cm.

- Use the correct midpoint or perpendicular-bisector property and retain cm.

**Independent practice** 7

P lies on the perpendicular bisector of AB. PA=3x+1 cm and PB=2x+5 cm. Find x and check both distances.

$$3x + 1 = 2x + 5$$




Geometry diagram; not drawn to scale except for construction tasks.

### Reference answer

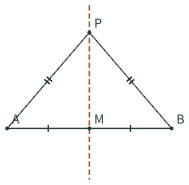
x=4; PA=PB=13 cm.

1. The perpendicular-bisector property gives PA=PB, so 3x+1=2x+5.
2. Subtract 2x and then subtract 1: x=4.
3. Check: 3×4+1=13 and 2×4+5=13; both are positive.

- Justify equal endpoint distances, solve x=4, and verify 13 cm.

**Independent practice** 8

P is off AB and PA=PB. M is the midpoint of AB. Prove PM⊥AB, and conclude that P lies on the perpendicular bisector of AB.




Geometry diagram; not drawn to scale except for construction tasks.

### Reference answer

△PMA≅△PMB by SSS; the equal adjacent angles at M total 180°, so each is 90°.

1. PA=PB is given, AM=BM because M is the midpoint, and PM is common.
2. Therefore △PMA≅△PMB by SSS.
3. Corresponding angles give ∠PMA=∠PMB.
4. A,M,B are collinear, so these adjacent angles total 180°; each is 90°.
5. Thus PM⊥AB through its midpoint, placing P on the perpendicular bisector.

- Supply all three SSS side pairs.
- Use both equal angles and their straight-angle sum.
- Conclude perpendicularity through the midpoint.
- Accept another complete proof if every condition and deduction is justified.

**Exit question** 9

A line passes through the midpoint of AB, but it is not stated to be perpendicular to AB. Must every point on it be equidistant from A and B?

A. No; the perpendicular condition is also needed.
B. Yes; midpoint information alone is enough.
C. Yes; all lines through one point coincide.
D. No line can be perpendicular to AB.

**Reference answer**

No. A perpendicular bisector requires both midpoint and perpendicularity.

1. Passing through the midpoint supplies the "bisector" part only.
2. Without perpendicularity, the equal-distance theorem does not apply to every point on the line.

- Identify the missing perpendicular requirement.

**Exit question** 10

A line is perpendicular to AB but crosses it away from its midpoint. Can it be the perpendicular bisector of AB?

A. No; perpendicular lines cannot intersect.
B. No; it fails the midpoint requirement.
C. Yes; perpendicularity alone is enough.
D. Yes; a segment has many different midpoints.

**Reference answer**

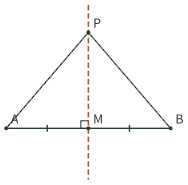
No. It must also pass through the unique midpoint.

1. Perpendicularity supplies one definition requirement.
2. The intersection must divide AB into two equal halves as well.

- Explain why the midpoint condition is indispensable.

**Further practice** 11

M is the midpoint of AB and AB=7.6 cm. Find AM.




Geometry diagram; not drawn to scale except for construction tasks.

**Reference answer**

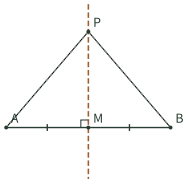
AM=3.8 cm.

1. The midpoint divides AB into two equal segments.
2. 7.6÷2=3.8, so AM=3.8 cm.

- Use the correct midpoint or perpendicular-bisector property and retain cm.

**Further practice** 12

P lies on the perpendicular bisector of AB. If PA=11.5 cm, find PB and state the property.




Geometry diagram; not drawn to scale except for construction tasks.

**Reference answer**

PB=11.5 cm: a point on the perpendicular bisector is equidistant from the endpoints.

1. The point is on the perpendicular bisector, so PA=PB.
2. Substitute PA=11.5 cm to obtain PB=11.5 cm.

- Use the correct midpoint or perpendicular-bisector property and retain cm.

**Further practice** 13

If A≠B and PA=PB, must P be inside some specified angle before the perpendicular-bisector converse can be used? Contrast this with the angle-bisector converse.

**Reference answer**

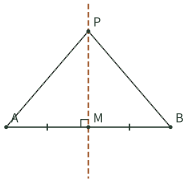
No specified-angle interior restriction is needed for a segment’ s perpendicular bisector. The stated internal angle-bisector converse does require P inside that angle.

1. PA and PB are distances to two endpoints of a segment.
2. Their equality locates P on the full perpendicular-bisector line on either side of AB.
3. Distances to angle sides involve a different theorem and its stated interior restriction.

- Distinguish the two geometric situations and their domains.

**Further practice** 14

To prove PA=PB from M being the midpoint and PM⊥AB, a pupil writes “PA=PB, therefore HL gives congruence.” Identify the circular step and give a valid criterion.




Geometry diagram; not drawn to scale except for construction tasks.

**Reference answer**

PA=PB was the target, not a given. Use AM=BM, common PM and the included right angles for SAS.

1. The pupil has assumed the desired equality of the hypotenuses.
2. The given facts instead supply equal half-segments and two included right angles.
3. Add common PM and apply SAS, then deduce PA=PB.

- Identify the assumed conclusion and replace it with justified SAS data.

**Extension** 15

A,B,C are noncollinear. P lies on the perpendicular bisectors of AB and BC. Prove that P also lies on the perpendicular bisector of AC.

**Reference answer**

PA=PB and PB=PC, so PA=PC; the converse places P on AC’ s perpendicular bisector.

1. The property for AB gives PA=PB.
2. The property for BC gives PB=PC.
3. Thus PA=PC, and A≠C because the vertices are noncollinear.
4. Use the endpoint-distance converse for AC.

- Use two forward properties followed by the correct converse.
- Accept another complete proof if every condition and deduction is justified.

**Extension** 16

M is the midpoint of AB. If P=M, the usual SAS triangles collapse. Is the equality PA=PB still true? Justify it without congruent triangles.

**Reference answer**

Yes: PA=MA and PB=MB, and MA=MB by the definition of midpoint.

1. Replace P by M in the two lengths.
2. The midpoint definition already gives MA=MB.
3. No congruence criterion is needed for this special case.

- Handle the degenerate case directly rather than invoking SAS on collapsed triangles.