Factorisation: common factors and square patterns — Teacher answers

Year 8, upper volume • text pp. 73–78 • foundation lessons 37–39

Use the worked model and guided attempt selectively, then the eight core questions. Keep further practice and extensions optional. Ask for a reason and an original-equation or expansion check.

Worked example 1

Factorise completely over the integers. Verify by expansion.

$$2x^3 - 18x$$

Reference answer

2x(x-3)(x+3).

$$2x(x-3)(x+3)$$

1. Extract 2x first; the remaining x^2-9 is a difference of squares.
2. $2x^3-18x = 2x(x^2-9)$.
3. The complete product is 2x(x-3)(x+3).
4. Expanding the product gives $2x^3-18x$, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.

Guided attempt 2

Factorise completely over the integers. Verify by expansion.

$$x^2 - 6x + 9$$

1. $9=___^2$; $-6x=-2 \cdot x \cdot ___$.
2. Therefore the product is $(___-___)^2$.

Reference answer

$(x-3)^2$.

$$(x-3)^2$$

1. Check the two end squares and the middle term $\pm 2ab$ before using $(a\pm b)^2$.
2. $x^2-6x+9 = x^2-2\times x\times 3+3^2$.
3. The complete product is $(x-3)^2$.
4. Expanding the product gives x^2-6x+9, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.

Prerequisite 3

Factorise completely over the integers. Verify by expansion.

$$6x^2 + 9x$$

Reference answer

3x(2x+3).

$$3x(2x+3)$$

1. Extract the common numerical factor and each common variable to its smallest exponent.
2. $6x^2+9x = 3x(2x+3)$.
3. The complete product is 3x(2x+3).
4. Expanding the product gives $6x^2+9x$, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.

Prerequisite 4

Factorise completely over the integers. Verify by expansion.

$$x^2 - 16$$

Reference answer

(x-4)(x+4).

$$(x - 4)(x + 4)$$

1. Recognise two squares with a subtraction, then write their sum times their difference.
2. x^2-16 = x^2-4^2.
3. The complete product is (x-4)(x+4).
4. Expanding the product gives x^2-16, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.

Independent practice 5

Factorise completely over the integers. Verify by expansion.

$$8x^2 - 12x$$

Reference answer

4x(2x-3).

$$4x(2x - 3)$$

1. Extract the common numerical factor and each common variable to its smallest exponent.
2. $8x^2$-12x = 4x(2x-3).
3. The complete product is 4x(2x-3).
4. Expanding the product gives $8x^2$-12x, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.

Independent practice 6

Factorise completely over the integers. Verify by expansion.

$$9x^2 - 25$$

Reference answer

(3x-5)(3x+5).

$$(3x - 5)(3x + 5)$$

1. Recognise two squares with a subtraction, then write their sum times their difference.
2. $9x^2$-25 = $(3x)^2$-5^2.
3. The complete product is (3x-5)(3x+5).
4. Expanding the product gives $9x^2$-25, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.

Independent practice 7

Factorise completely over the integers. Verify by expansion.

$$x^2 + 10x + 25$$

Reference answer

$(x+5)^2$.

$$(x + 5)^2$$

1. Check the two end squares and the middle term $\pm 2ab$ before using $(a \pm b)^2$.
2. $x^2+10x+25 = x^2+2\times x\times 5+5^2$.
3. The complete product is $(x+5)^2$.
4. Expanding the product gives $x^2+10x+25$, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.

Independent practice 8

Factorise completely over the integers. Verify by expansion.

$$3x^2 - 12$$

Reference answer

$3(x-2)(x+2)$.

$$3(x - 2)(x + 2)$$

1. Extract 3, then factor x^2-4 by difference of squares.
2. $3x^2-12 = 3(x^2-4)$.
3. The complete product is $3(x-2)(x+2)$.
4. Expanding the product gives $3x^2-12$, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.

Exit question 9

Factorise completely over the integers. Verify by expansion.

$$x^2 - 4x + 4$$

Reference answer

$(x-2)^2$.

$$(x - 2)^2$$

1. Check the two end squares and the middle term $\pm 2ab$ before using $(a \pm b)^2$.
2. $x^2-4x+4 = x^2-2\times x\times 2+2^2$.
3. The complete product is $(x-2)^2$.
4. Expanding the product gives x^2-4x+4, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.

Exit question 10

Factorise completely over the integers. Verify by expansion.

$$2x^2 + 6x$$

Reference answer

2x(x+3).

$$2x(x + 3)$$

1. Extract the common numerical factor and each common variable to its smallest exponent.
2. $2x^2+6x = 2x(x+3)$.
3. The complete product is 2x(x+3).
4. Expanding the product gives $2x^2+6x$, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.

Further practice 11

Factorise completely over the integers. Verify by expansion.

$$y^2 - 49$$

Reference answer

(y-7)(y+7).

$$(y - 7)(y + 7)$$

1. Recognise two squares with a subtraction, then write their sum times their difference.
2. $y^2-49 = y^2-7^2$.
3. The complete product is (y-7)(y+7).
4. Expanding the product gives y^2-49, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.

Further practice 12

Factorise completely over the integers. Verify by expansion.

$$4x^2 + 12x + 9$$

Reference answer

$(2x+3)^2$.

$$(2x + 3)^2$$

1. Check the two end squares and the middle term $\pm 2ab$ before using $(a\pm b)^2$.
2. $4x^2+12x+9 = (2x)^2+2\times(2x)\times 3+3^2$.
3. The complete product is $(2x+3)^2$.
4. Expanding the product gives $4x^2+12x+9$, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.

Further practice 13

Factorise completely over the integers. Verify by expansion.

$$(x+1)^2 - 9$$

Reference answer

(x-2)(x+4).

$$(x-2)(x+4)$$

1. Recognise two squares with a subtraction, then write their sum times their difference.
2. $(x+1)^2-9 = ((x+1)-3)((x+1)+3)$.
3. The complete product is (x-2)(x+4).
4. Expanding the product gives $(x+1)^2-9$, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.

Further practice 14

Factorise completely over the integers. Verify by expansion.

$$a^2b - ab^2$$

Reference answer

ab(a-b).

$$ab(a-b)$$

1. Extract the common numerical factor and each common variable to its smallest exponent.
2. $a^2b-ab^2 = ab(a-b)$.
3. The complete product is ab(a-b).
4. Expanding the product gives a^2b-ab^2, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.

Extension 15

Factorise completely over the integers. Verify by expansion.

$$x^4 - 16$$

Reference answer

$(x-2)(x+2)(x^2+4)$.

$$(x-2)(x+2)(x^2+4)$$

1. Apply difference of squares twice. Over integer coefficients, x^2+4 has no further linear factors.
2. $x^4-16 = (x^2-4)(x^2+4)$.
3. The complete product is $(x-2)(x+2)(x^2+4)$.
4. Expanding the product gives x^4-16, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.

Extension 16

Factorise completely over the integers. Verify by expansion.

$$8x^3 - 32x$$

Reference answer

8x(x-2)(x+2).

$$8x(x-2)(x+2)$$

1. Extract 8x, then finish the difference-of-squares factor.
2. $8x^3$-32x = 8x(x^2-4).
3. The complete product is 8x(x-2)(x+2).
4. Expanding the product gives $8x^3$-32x, the original expression.

- Finish as a product of integer-coefficient factors.
- Continue factoring when possible and verify the expansion.