Variance: calculation, stability and transformations — Teacher answers

Year 8, lower volume • text pp. 83–84 • textbook lesson 42

Use the two prerequisite checks to choose support. Project the worked and guided tasks, then collect the eight paper core questions. Follow-ups and extensions are optional; accept other complete valid methods. This page does not save learners' paper answers.

Ordinary route: prerequisites 4, short inquiry 5, worked/guided 8, independent 15, correction 5, exit 3 minutes. Advanced route: inquiry 18, organise 4, selected practice 13, exit 5 minutes.

Prerequisites: Mean, signed differences, squares and fractions.

Ordinary route: use the first fixed case for the five-minute demonstration. Advanced route: complete the full investigation.

1. 3′ Predict which is more spread out: A={4,5,6} or B={1,5,9}, both with mean 5.
2. 8′ On paper compute every deviation and squared deviation. Divide by n=3: variances $\frac{2}{3}$ and $\frac{32}{3}$. Add 3 to every value of A, then separately double every original value of A. The existing outlier engine does not calculate variance.
3. 5′ Explain unchanged variance after adding 3 and variance $\frac{8}{3}$ after doubling, four times the original. State squared units and retain the course convention of dividing by n.
4. 2′ Individually state how multiplying every value by −2 changes the variance, with a squared-deviation reason.

Worked example 1

Find the descriptive variance of $\{2, 4, 4, 6\}$, using the number of observations as the denominator. Show the mean first.

Reference answer

$s^2 = 2$

1. Mean=(2+4+4+6)/4=4.
2. Squared deviations are 4,0,0,4; their sum is 8.
3. Variance=8/4=2.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Guided attempt 2

Find the descriptive variance of $\{1, 3, 5\}$, using the number of observations as the denominator. Show the mean first.

1. Mean = $\frac{1+3+5}{3} = \square$.
2. Deviations: $1 - \square, \quad 3 - \square, \quad 5 - \square$.
3. Squared total = $(-2)^2 + 0^2 + 2^2 = \square$; variance = $\frac{\square}{3}$.

Reference answer

$s^2 = \frac{8}{3}$

1. Mean=3; deviations are −2,0,2.
2. The square total is 8, divided by 3 observations.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Prerequisite 3

Find the mean of $\{2, 4\}$. Show the relevant ordering or calculation.

Reference answer

3

1. Add every observation, then divide by the number of observations.
2. The result is 3.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Prerequisite 4

The reference mean is 5. Find the sum of squared deviations for 3,5,7.

Reference answer

8.

$$8$$

1. The deviations are −2,0,2 and their squares are 4,0,4.
2. Add the squares: 8.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 5

Find the descriptive variance of $\{1, 1, 3, 3\}$, using the number of observations as the denominator. Show the mean first.

Reference answer

$s^2 = 1$

1. Compute the mean from every observation.
2. Square each deviation from the mean, add, and divide by n.
3. The variance is 1; its unit is the square of the data unit.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 6

Find the descriptive variance of $\{2, 2, 2\}$, using the number of observations as the denominator. Show the mean first.

Reference answer

$s^2 = 0$

1. Compute the mean from every observation.
2. Square each deviation from the mean, add, and divide by n.
3. The variance is 0; its unit is the square of the data unit.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 7

Group A is 4,5,6 and group B is 3,5,7. Check both means, compare both n-denominator variances, and decide which set is more stable on the same scale.

Reference answer

Both means are 5. A has variance 2/3, B has 8/3, so A is less variable.

1. The two means agree, so compare spread rather than centre.
2. Square totals are 2 and 8, and both sets contain 3 values.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Independent practice 8

Find the descriptive variance of $\{0, 2, 4\}$, using the number of observations as the denominator. Show the mean first.

Reference answer

$s^2 = \frac{8}{3}$

1. Compute the mean from every observation.
2. Square each deviation from the mean, add, and divide by n.
3. The variance is $8/3$; its unit is the square of the data unit.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Exit question 9

Find the descriptive variance of $\{1, 3\}$, using the number of observations as the denominator. Show the mean first.

Reference answer

$s^2 = 1$

1. Compute the mean from every observation.
2. Square each deviation from the mean, add, and divide by n.
3. The variance is 1; its unit is the square of the data unit.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Exit question 10

Find the descriptive variance of $\{5, 5, 5, 5\}$, using the number of observations as the denominator. Show the mean first.

Reference answer

$s^2 = 0$

1. Compute the mean from every observation.
2. Square each deviation from the mean, add, and divide by n.
3. The variance is 0; its unit is the square of the data unit.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 11

The data 2,4,4,6 have variance 2. Add 3 to every value. Find the new variance and explain the unchanged deviations.

Reference answer

$s^2 = 2$

1. The mean also increases by 3, from 4 to 7.
2. Every deviation remains −2,0,0,2; the variance remains 2.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 12

The data 2,4,4,6 have variance 2. Multiply every value by 2. Find the new variance and explain the scale factor.

Reference answer

$s^2 = 8$

1. Each deviation doubles, so each squared deviation is multiplied by 4.
2. The variance becomes 4×2=8.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 13

Find the descriptive variance of $\{-2, 0, 2\}$, using the number of observations as the denominator. Show the mean first.

Reference answer

$s^2 = \frac{8}{3}$

1. Compute the mean from every observation.
2. Square each deviation from the mean, add, and divide by n.
3. The variance is $8/3$; its unit is the square of the data unit.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Further practice 14

Values 1 and 3 each occur twice. Find the mean and n-denominator variance, counting all four observations.

Value	Frequency / weight
1	2
3	2

Use the stated conditions and the labelled scales.

Reference answer

Mean 2 and variance 1.

1. The data are 1,1,3,3; the mean is 2.
2. The squared-deviation total is 4 over 4 observations.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Extension 15

The data 1,1,5,5 have mean 3. Find their variance and explain why knowing the mean alone does not determine spread.

Reference answer

$s^2 = 4$

1. Every deviation is −2 or 2, so every squared deviation is 4.
2. Variance=16/4=4; data 3,3,3,3 have the same mean but zero variance.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.

Extension 16

For this chapter’ s descriptive variance of 2,4, a learner divides the squared-deviation total 2 by n−1 and reports 2. Correct the calculation under the stated n-denominator definition.

Reference answer

$s^2 = 1$

1. The mean is 3 and the square total is 2.
2. Here n=2, so the descriptive variance is 2/2=1. A different statistical estimator uses another definition.

- Use the stated conditions and justify the relation before calculating.
- Give an exact answer where possible, with units and any domain restrictions.