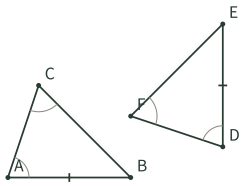
Congruence: ASA, AAS and right-triangle HL — Teacher answers

Year 8, upper volume • text pp. 19–24 • foundation lessons 10–12

Use the worked model and guided attempt selectively, then the eight core questions. Keep further practice and extensions optional. Ask for a reason and an original-equation or expansion check.

Worked example 1

Use the two marked angle pairs and AB=DE to prove congruence by AAS.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

△ABC≅△DEF by AAS, with A↔D, B↔E and C↔F.

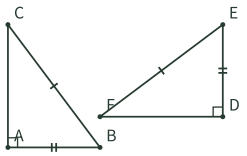
1. ∠BAC=∠EDF and ∠ACB=∠DFE are given.
2. AB=DE is not the side between those angles.
3. The two angle pairs and this corresponding side give △ABC≅△DEF by AAS.

- Use only given or independently proved facts.
- Keep the vertex order consistent. Other complete justified proofs are accepted.

Guided attempt 2

Both triangles have the marked right angles. Complete an HL proof using the marked side pairs.

1. The hypotenuses are ___ and ___.
2. Equal hypotenuses ___=___ and legs ___=___ give ___.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

△ABC≅△DEF by HL, with A↔D, B↔E and C↔F.

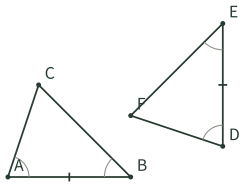
1. ∠BAC=∠EDF=90°, so both triangles are right triangles.
2. BC=EF are the hypotenuses, and AB=DE are a pair of corresponding legs.
3. Thus △ABC≅△DEF by HL.

- Use only given or independently proved facts.
- Keep the vertex order consistent. Other complete justified proofs are accepted.

Prerequisite 3

Which side lies between ∠BAC and ∠ABC?

A. BC
B. There is no included side.
C. AB
D. AC



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

AB

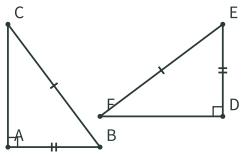
1. The angle vertices are A and B.
2. The side joining them is AB.

- Choose the statement justified by the stated conditions and explain why.

Prerequisite 4

∠BAC=90°. Which side is the hypotenuse of △ABC?

A. AB
B. AC
C. Any side may be the hypotenuse.
D. BC



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

BC

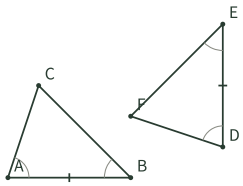
1. The hypotenuse is opposite the right angle.
2. The side opposite A is BC.

- Choose the statement justified by the stated conditions and explain why.

Independent practice 5

The equal angles are at A,D and B,E, with AB=DE. Which criterion uses exactly these facts directly?

A. ASA
B. SSS
C. HL
D. AAA



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

ASA

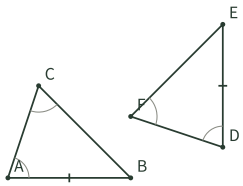
1. ∠BAC=∠EDF and ∠ABC=∠DEF are given.
2. AB=DE is the side between those two angles.
3. Thus △ABC≅△DEF by ASA.

- Choose the statement justified by the stated conditions and explain why.

Independent practice 6

The equal angles are at A,D and C,F, with AB=DE. Which direct criterion applies?

A. SSA
B. AAS
C. ASA using AB as the included side
D. SSS



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

AAS

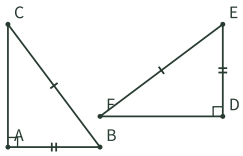
1. ∠BAC=∠EDF and ∠ACB=∠DFE are given.
2. AB=DE is not the side between those angles.
3. The two angle pairs and this corresponding side give △ABC≅△DEF by AAS.

- Choose the statement justified by the stated conditions and explain why.

Independent practice 7

In the marked right triangles, BC=EF and AB=DE. Which conclusion follows?

A. All triangle angles are 90°.
B. The information is AAA.
C. AC=DF by HL congruence.
D. AC=DE because the drawing looks equal.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

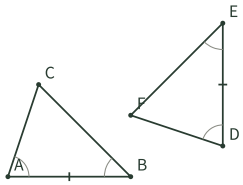
AC=DF by HL congruence.

1. ∠BAC=∠EDF=90°, so both triangles are right triangles.
2. BC=EF are the hypotenuses, and AB=DE are a pair of corresponding legs.
3. Thus △ABC≅△DEF by HL.
4. Corresponding remaining legs give AC=DF.

- Choose the statement justified by the stated conditions and explain why.

Independent practice 8

Use the given two angle pairs and their included side to prove congruence. Name your criterion and reasons.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

△ABC≅△DEF by ASA, with A↔D, B↔E and C↔F.

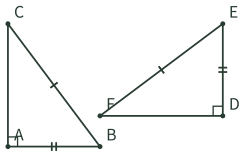
1. ∠BAC=∠EDF and ∠ABC=∠DEF are given.
2. AB=DE is the side between those two angles.
3. Thus △ABC≅△DEF by ASA.

- Use only given or independently proved facts.
- Keep the vertex order consistent. Other complete justified proofs are accepted.

Exit question 9

Why does an HL proof need both right-angle conditions before identifying the hypotenuses?

A. HL is a criterion for two right triangles; a longest-looking side is insufficient.
B. Any two triangles have hypotenuses.
C. The right angles may be assumed from the picture.
D. HL only needs one arbitrary equal side.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

HL is a criterion for two right triangles; a longest-looking side is insufficient.

1. ∠BAC=∠EDF=90°, so both triangles are right triangles.
2. BC=EF are the hypotenuses, and AB=DE are a pair of corresponding legs.
3. Thus △ABC≅△DEF by HL.

- Choose the statement justified by the stated conditions and explain why.

Exit question 10

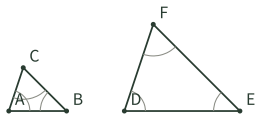
Can three equal angle pairs replace the equal-side requirement in ASA or AAS?

A. Yes if the angles are acute.

B. No: scale is not fixed without a corresponding side.

C. Yes: AAA is stronger than ASA.

D. Yes if the triangles are labelled.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

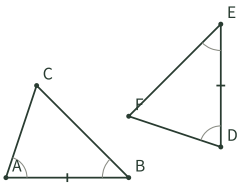
No: scale is not fixed without a corresponding side.

1. All corresponding angles can stay equal under enlargement.
2. The displayed counterexample has doubled sides and is not congruent.

- Choose the statement justified by the stated conditions and explain why.

Further practice 11

Write the two equal-angle pairs, their included side and the ASA conclusion.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

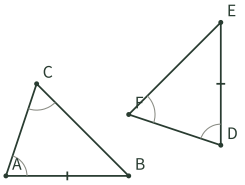
$\triangle ABC \cong \triangle DEF$ by ASA, with A↔D, B↔E and C↔F.

1. $\angle BAC = \angle EDF$ and $\angle ABC = \angle DEF$ are given.
2. AB=DE is the side between those two angles.
3. Thus $\triangle ABC \cong \triangle DEF$ by ASA.

- Use only given or independently proved facts.
- Keep the vertex order consistent. Other complete justified proofs are accepted.

Further practice 12

Write an AAS proof and explain why AB is a non-included side here.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

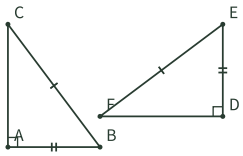
$\triangle ABC \cong \triangle DEF$ by AAS, with A↔D, B↔E and C↔F.

1. $\angle BAC = \angle EDF$ and $\angle ACB = \angle DFE$ are given.
2. AB=DE is not the side between those angles.
3. The two angle pairs and this corresponding side give $\triangle ABC \cong \triangle DEF$ by AAS.

- Use only given or independently proved facts.
- Keep the vertex order consistent. Other complete justified proofs are accepted.

Further practice 13

List the right angles, hypotenuses and equal legs before the HL conclusion.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

△ABC≅△DEF by HL, with A↔D, B↔E and C↔F.

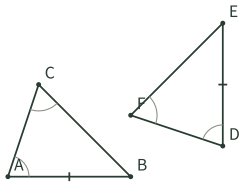
1. ∠BAC=∠EDF=90°, so both triangles are right triangles.
2. BC=EF are the hypotenuses, and AB=DE are a pair of corresponding legs.
3. Thus △ABC≅△DEF by HL.

- Use only given or independently proved facts.
- Keep the vertex order consistent. Other complete justified proofs are accepted.

Further practice 14

After the AAS proof, which additional angle equality follows?

A. ∠ABC=90°
B. ∠ABC=∠DEF
C. ∠ABC=∠DFE
D. ∠ABC=∠EDF



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

∠ABC=∠DEF

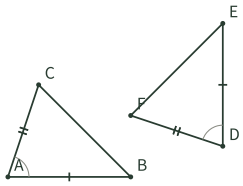
1. ∠BAC=∠EDF and ∠ACB=∠DFE are given.
2. AB=DE is not the side between those angles.
3. The two angle pairs and this corresponding side give △ABC≅△DEF by AAS.
4. B corresponds to E, so the remaining angle equality is ∠ABC=∠DEF.

- Choose the statement justified by the stated conditions and explain why.

Extension 15

The marked two side pairs and their included angle already give SAS. Which extra side equality would ALSO allow a direct SSS proof?

A. The triangles have equal areas only.
B. The shapes look the same.
C. BC=EF
D. ∠ABC=∠DEF only



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

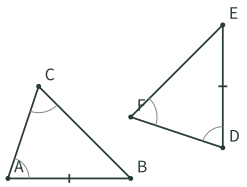
BC=EF

1. The existing two side pairs and included angle already justify SAS.
2. A direct SSS proof additionally needs the third corresponding side pair BC=EF.
3. With that extra equality, AB=DE, AC=DF and BC=EF also give △ABC≅△DEF by SSS.

- Choose the statement justified by the stated conditions and explain why.

Extension 16

Give an alternative proof of the marked AAS result: first obtain the third angle pair, then use ASA.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

△ABC≅△DEF by AAS, with A↔D, B↔E and C↔F.

1. The two given angle pairs are equal. Subtract each pair from 180° to obtain ∠ABC=∠DEF.
2. Now ∠BAC=∠EDF, ∠ABC=∠DEF and the included side AB=DE give ASA congruence.

- Use only given or independently proved facts.
- Keep the vertex order consistent. Other complete justified proofs are accepted.