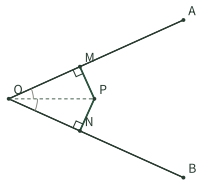
Angle bisectors: perpendicular distances and the forward proof — Teacher answers

Year 8, upper volume • text pp. 25–26 • foundation lesson 13

Use a short projected observation, one complete AAS model, and a fill-in attempt before independent work. Never introduce PM=PN as a given in the forward proof. Accept other justified proofs.

Worked example 1

OP bisects ∠AOB. P is inside the angle, PM⊥OA and PN⊥OB. Prove PM=PN using congruent triangles.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

△OPM≅△OPN by AAS, so PM=PN.

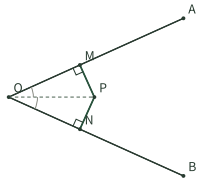
1. ∠OMP=∠ONP=90° because the two segments are perpendicular to the sides.
2. ∠MOP=∠PON by the definition of an angle bisector.
3. OP=OP, the common side. It is opposite the right angle in each triangle.
4. Therefore △OPM≅△OPN by AAS.
5. Hence PM=PN, since corresponding sides of congruent triangles are equal.

- State the right angles and their perpendicular-line reason.
- State the equal vertex angles and the common side.
- Use a valid congruence criterion without assuming the conclusion.
- Keep vertex correspondence consistent and deduce equal distances.
- Accept another complete proof if every condition and deduction is justified.

Guided attempt 2

OP bisects ∠AOB, P is inside the angle, PM⊥OA and PN⊥OB. Complete the proof table without using PM=PN as an input.

1. ∠OMP=∠___=90° because ___.
2. ∠MOP=∠___ because ___.
3. ___=___ is common, so △OPM≅△OPN by ___.
4. Therefore ___=___, by ___.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

Right angles OMP and ONP; ∠MOP=∠PON; OP=OP; AAS; PM=PN.

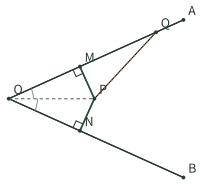
1. ∠OMP=∠ONP=90° because the two segments are perpendicular to the sides.
2. ∠MOP=∠PON by the definition of an angle bisector.
3. OP=OP, the common side. It is opposite the right angle in each triangle.
4. Therefore △OPM≅△OPN by AAS.
5. Hence PM=PN, since corresponding sides of congruent triangles are equal.

- State the right angles and their perpendicular-line reason.
- State the equal vertex angles and the common side.
- Use a valid congruence criterion without assuming the conclusion.
- Keep vertex correspondence consistent and deduce equal distances.
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Prerequisite 3

M,Q lie on OA. PM⊥OA, while PQ is oblique. Which segment measures the distance from P to OA?

A. PQ
B. OM
C. PM
D. OP



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

PM: the perpendicular segment from the point to the side.

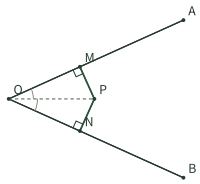
1. Distance from a point to a line means the perpendicular length.
2. PM is perpendicular to OA; OP ends at the vertex and PQ is oblique.

- Select PM and explain the perpendicular condition.

Prerequisite 4

OP bisects ∠AOB=64°. Find ∠AOP.

A. 64°
B. 90°
C. 128°
D. 32°



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

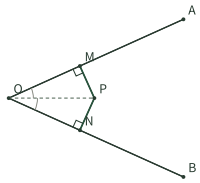
32°.

1. The two parts have equal angles.
2. 64°÷2=32°.

- Halve the whole angle and retain the degree unit.

Independent practice 5

P lies on the internal bisector of ∠AOB; PM⊥OA and PN⊥OB. If PM=6.4 cm, find PN and name the property.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

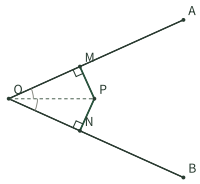
PN=6.4 cm, by the angle-bisector property.

1. Both lengths are perpendicular distances from the same point.
2. The point lies on the bisector, so PN=PM=6.4 cm.

- Use perpendicular lengths, not an oblique segment or distance to the vertex.
- Give the value with the stated unit and name the property.

Independent practice 6

OP is the internal bisector of ∠AOB=74°. Find ∠PON.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

∠PON=37°.

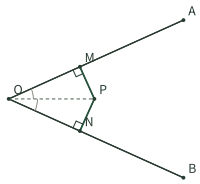
1. ∠AOP=∠PON by the definition of bisector.
2. Each is 74°÷2=37°.

- Give 37° with the bisector reason.

Independent practice 7

P lies on the internal bisector; PM⊥OA and PN⊥OB. PM=2x+1 cm and PN=3x−2 cm. Find x and check that both lengths are positive.

$$2x + 1 = 3x - 2$$



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

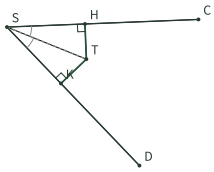
x=3; both lengths are 7 cm.

1. Set PM=PN by the property: 2x+1=3x−2.
2. Subtract 2x and add 2: x=3.
3. Substitution gives 2×3+1=3×3−2=7>0.

- Justify the equation using perpendicular distances.
- Solve x=3 and verify both lengths are 7 cm.

Independent practice 8

ST bisects ∠CSD and T is inside the angle. TH⊥SC and TK⊥SD. Prove TH=TK; choose the triangles yourself.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

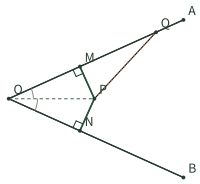
△STH≅△STK by AAS; hence TH=TK.

1. ∠SHT=∠SKT=90° because the two segments are perpendicular to the sides.
2. ∠HST=∠TSK by the definition of an angle bisector.
3. ST=ST, the common side. It is opposite the right angle in each triangle.
4. Therefore △STH≅△STK by AAS.
5. Hence TH=TK, since corresponding sides of congruent triangles are equal.

- State the right angles and their perpendicular-line reason.
- State the equal vertex angles and the common side.
- Use a valid congruence criterion without assuming the conclusion.
- Keep vertex correspondence consistent and deduce equal distances.
- Accept another complete proof if every condition and deduction is justified.

Exit question 9

P lies on the angle bisector and PN⊥OB. Someone gives PQ=3 cm, where Q lies on OA but PQ is not perpendicular to OA. Must PN=3 cm? Explain.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

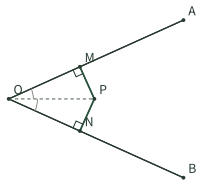
No. PQ is oblique and does not measure the distance to OA. The property equates perpendicular distances only.

1. Check the meaning of the given length before applying the property.
2. PQ is not a perpendicular distance, so its length cannot determine PN this way.

- Reject the conclusion and identify the missing perpendicular condition.

Exit question 10

A lamp P stands inside the angle formed by two straight paths, on their internal angle bisector. Its shortest distance to the first path is 9 m. Find its shortest distance to the second path.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

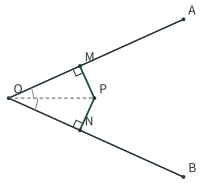
9 m: shortest distances are perpendicular distances, and the point is on the bisector.

1. Interpret each shortest distance as a perpendicular length.
2. Use the angle-bisector property to transfer 9 m to the second path.

- Use perpendicular lengths, not an oblique segment or distance to the vertex.
- Give the value with the stated unit and name the property.

Further practice 11

P lies on the internal bisector, with PM⊥OA and PN⊥OB. PM=3.75 cm. Find PN.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

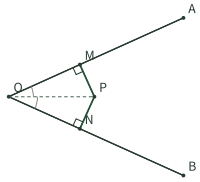
PN=3.75 cm.

1. PM and PN are the perpendicular distances from P.
2. Hence PN=PM=3.75 cm.

- Use perpendicular lengths, not an oblique segment or distance to the vertex.
- Give the value with the stated unit and name the property.

Further practice 12

P lies on the internal bisector; PM⊥OA and PN⊥OB. PM=3x−1 cm and PN=8 cm. Find x.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

x=3; both perpendicular lengths are 8 cm.

1. Set 3x−1=8.
2. Then 3x=9, so x=3.
3. Check 3×3−1=8>0.

- Link the equation to the property and obtain x=3.

Further practice 13

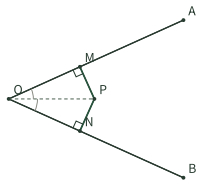
To prove PM=PN from OP bisecting the angle and the two perpendicular conditions, which starting data are legitimate?

A. Two right angles, equal angles at O, and OP common.

B. PM=PN, so HL applies immediately.

C. Three equal angles prove congruence by AAA.

D. Assume OM=ON without justification.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

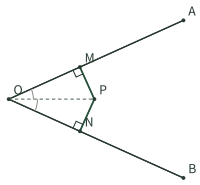
Use the right angles, the bisected angles, and common OP; AAS proves the result.

1. The perpendicular and bisector conditions supply the two angle pairs.
2. OP is shared. PM=PN is the target, not a premise.

- Select only given or independently justified facts.
- Explain why assuming PM=PN would be circular.

Further practice 14

P is on the internal bisector, PM⊥OA and PN⊥OB. PM=6 cm and △OPM has perimeter 24 cm. Find OP+ON.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

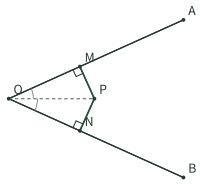
OP+ON=18 cm.

1. The AAS congruence also gives OM=ON.
2. OP+OM=24−6=18 cm.
3. Replace OM by ON to obtain OP+ON=18 cm.

- Justify OM=ON and subtract the correct side.

Extension 15

Three trials show the same rounded values for PM and PN. Explain what this supports, and what is still needed for every point on the bisector.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

The trials support the conjecture. A general congruence proof, not repeated or rounded measurements, is needed for all such points.

1. A finite set of trials covers only those settings.
2. Rounded equality may conceal a small difference.
3. The AAS proof uses the stated conditions for an arbitrary eligible point.

- Distinguish conjectural evidence from a general proof.

Extension 16

A pupil says every point inside an angle has equal distances to the sides. A point Q inside the angle has perpendicular distances 2 cm and 4 cm. Is it a counterexample, and what condition repairs the claim?

Reference answer

Yes: $2 \neq 4$. Restrict the point to the internal angle bisector.

1. Q satisfies the proposed premise, being inside the angle.
2. Its perpendicular distances differ, so the proposed conclusion fails.
3. The forward theorem requires the point to be on the bisector.

- Verify both parts of the counterexample and state the missing bisector condition.