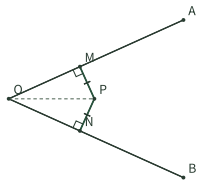
Angle bisectors: the converse and applications — Teacher answers

Year 8, upper volume • text pp. 27–28 • foundation lesson 14

Model HL with OP as the hypotenuse, then ask pupils to label given and required facts separately. Make the interior and perpendicular conditions visible in each proof.

Worked example 1

P is inside ∠AOB. PM⊥OA, PN⊥OB and PM=PN. Prove that OP bisects ∠AOB.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

The right triangles are congruent by HL, so the vertex angles match and OP is the internal bisector.

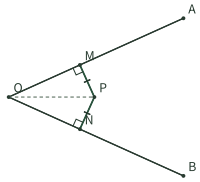
1. The perpendicular conditions make △OPM and △OPN right triangles.
2. OP=OP is the common hypotenuse, and PM=PN is given.
3. Thus △OPM≅△OPN by HL.
4. Corresponding angles give ∠MOP=∠PON.
5. Since P is inside the angle, OP is its internal bisector.

- State both right triangles and identify the common hypotenuse.
- Use the given equal legs and HL.
- Deduce equal vertex angles and retain the interior condition.
- Accept another complete proof if every condition and deduction is justified.

Guided attempt 2

P is inside ∠AOB, PM⊥OA, PN⊥OB and PM=PN. Complete the HL proof without assuming any equal angles at O.

1. The two right angles are at ___ and ___.
2. Hypotenuse ___=___; leg ___=___ is given.
3. △OPM≅△OPN by ___, so ∠___=∠___.
4. Because P is ___, OP is the ___ bisector.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

OP is the common hypotenuse; PM=PN are equal legs; HL gives ∠MOP=∠PON, hence the internal bisector.

1. The perpendicular conditions make △OPM and △OPN right triangles.
2. OP=OP is the common hypotenuse, and PM=PN is given.
3. Thus △OPM≅△OPN by HL.
4. Corresponding angles give ∠MOP=∠PON.
5. Since P is inside the angle, OP is its internal bisector.

- State both right triangles and identify the common hypotenuse.
- Use the given equal legs and HL.
- Deduce equal vertex angles and retain the interior condition.
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Prerequisite 3

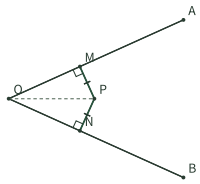
PM⊥OA and M lies on OA. Which side is the hypotenuse of △OPM?

A. OM

B. Any side can be called the hypotenuse.

C. OP

D. PM



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

OP, because it is opposite the right angle at M.

1. PM⊥OM gives a right angle at M.
2. The side opposite M is OP.

- Identify OP by its position opposite the right angle.

Prerequisite 4

Which statement is the converse of “a point on the internal bisector has equal perpendicular distances to the sides”?

A. Every interior point has equal distances to the sides.

B. A point on a side is the angle vertex.

C. Two equal angles always have equal side lengths.

D. An interior point with equal perpendicular distances lies on the internal bisector.

Reference answer

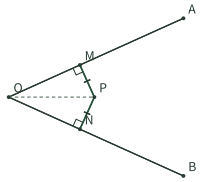
The first statement, retaining the interior and perpendicular-distance conditions.

1. Exchange the original premise and conclusion.
2. Keep the domain restriction: P must be inside the angle.

- Reverse the logical direction without deleting necessary conditions.

Independent practice 5

P is inside ∠AOB=86°. PM⊥OA, PN⊥OB and PM=PN=4 cm. Find ∠AOP and give the reason for halving.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

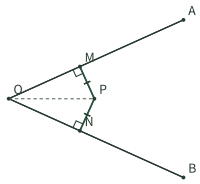
43°: the converse first proves that OP bisects the angle.

1. The equal perpendicular distances and interior condition give the internal bisector.
2. Thus ∠AOP=86°÷2=43°.

- State the converse before the numerical halving.

Independent practice 6

P is inside ∠AOB. PM⊥OA, PN⊥OB and PM=PN. If ∠MOP=28°, find ∠AOB.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

∠AOB=56°.

1. The converse gives ∠MOP=∠PON.
2. The whole angle is 28°+28°=56°.

- Justify equal angles and sum them correctly.

Independent practice 7

Distances to the two intersecting lines containing OA and OB are equal, but P is not stated to be inside ∠AOB. Can we conclude that P lies on this angle’s internal bisector?

A. Yes, because all intersecting lines are perpendicular.
B. No point can have two equal distances.
C. Not necessarily; the interior condition is needed for that conclusion.
D. Yes, with no position condition.

Reference answer

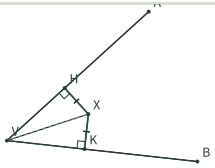
Not necessarily. Equal distances to two full lines also occur on the other bisector.

1. The proposed conclusion concerns a particular internal angle.
2. An equidistant point outside that angle need not lie on its internal bisector.

- Explain the role of the interior restriction.

Independent practice 8

X is inside ∠AVB. XH⊥VA, XK⊥VB and XH=XK. Prove that VX is the internal angle bisector.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

△VXH≅△VXK by HL; their equal angles at V make VX the internal bisector.

1. The perpendicular conditions make △VXH and △VXK right triangles.
2. VX=VX is the common hypotenuse, and XH=XK is given.
3. Thus △VXH≅△VXK by HL.
4. Corresponding angles give ∠HVX=∠XVK.
5. Since X is inside the angle, VX is its internal bisector.

- State both right triangles and identify the common hypotenuse.
- Use the given equal legs and HL.
- Deduce equal vertex angles and retain the interior condition.
- Accept another complete proof if every condition and deduction is justified.

Exit question 9

P is inside ∠AOB. M lies on OA, N lies on OB and PM=PN. No perpendicular conditions are given. Is this enough to prove that OP is the bisector?

Reference answer

No. PM and PN need not be the perpendicular distances to the sides.

1. The converse compares distances, meaning perpendicular lengths.
2. Equal arbitrary segments ending on the sides do not supply that premise.

- Reject the inference and name the missing perpendicular facts.

Exit question 10

P is inside the angle and its two perpendicular distances are equal. Which result directly tells us where P lies?

A. Equality of all three angles, without side data.
B. The converse of the angle-bisector property.
C. The forward property, starting by assuming OP bisects the angle.
D. The triangle perimeter formula alone.

Reference answer

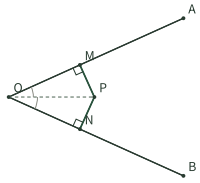
The converse places P on the internal angle bisector.

1. Equal perpendicular distances are the given facts.
2. The desired conclusion is the bisector, so use the converse.

- Choose the direction that matches the given facts.

Further practice 11

P lies on the internal bisector, PM⊥OA and PN⊥OB. PM=3 cm and △OPM has perimeter 12 cm. Find OP+ON.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

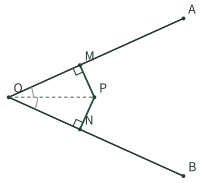
OP+ON=9 cm.

1. The right triangles are congruent, giving OM=ON.
2. OP+ON=OP+OM=12−3=9 cm.

- Use OM=ON and subtract the given leg.

Further practice 12

P lies on the internal bisector, PM⊥OA and PN⊥OB. PM=9 cm and △OPM has perimeter 36 cm. Find OP+ON.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

OP+ON=27 cm.

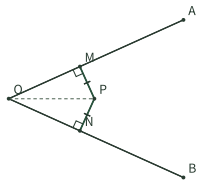
1. Congruence gives OM=ON.
2. The two requested sides total 36−9=27 cm.

- Identify the correct corresponding side and total.

Further practice 13

P is inside the angle, PM⊥OA, PN⊥OB and PM=PN=5 cm. Which conclusion follows directly?

A. OP bisects the angle; its length is not determined by 5 cm alone.
B. OP=5 cm.
C. The whole angle must be 90°.
D. M=N.



Geometry diagram; not drawn to scale except for construction tasks.

Reference answer

The first conclusion: the converse determines the direction, not OP’ s length from one distance alone.

1. Equal perpendicular distances satisfy the converse.
2. Different angle openings can have that same perpendicular distance but different OP lengths.

- State the bisector conclusion without inventing a length or angle.

Further practice 14

Complete two sentences: forward direction, given ___, prove equal perpendicular distances; converse, given ___, prove the internal bisector. Include all position and perpendicular conditions.

Reference answer

Forward: an interior point on the bisector, with perpendicular feet. Converse: an interior point with equal perpendicular distances to the sides.

1. In the forward proof, the bisector gives equal vertex angles.
2. In the converse, equal legs and the common hypotenuse give HL.
3. Keep “inside” and “perpendicular distance” in the converse.

- Give both directions without circular assumptions.

Extension 15

I is inside △ABC and lies on the internal bisectors at A and B. Let D,E,F be its perpendicular feet on AB,AC,BC respectively. Prove that I also lies on the internal bisector at C.

Reference answer

ID=IE and ID=IF, hence IE=IF. The interior converse at C places I on C’ s bisector.

1. The bisector at A gives ID=IE.
2. The bisector at B gives ID=IF.
3. Therefore IE=IF. These are perpendicular distances to CA and CB.
4. I is inside ∠ACB, so its converse gives the third bisector.

- Use both forward properties and the final converse with the interior condition.
- Accept another complete proof if every condition and deduction is justified.

Extension 16

Someone puts P=O, obtains two zero distances, and tries to use HL to define the ray OP as an angle bisector. Explain why this proof setup fails.

Reference answer

OP is not a ray determined by two distinct points, and both supposed triangles are degenerate. The theorem’ s interior-point setup excludes the vertex.

1. A ray OP requires P≠O.
2. With P=O there are no nondegenerate right triangles for HL.
3. Use the interior-point hypothesis as stated.

- Identify both the undefined ray and degenerate-triangle problems.