Three-variable systems: eliminate, solve and check — Teacher answers

Year 7, lower volume • text pp. 67–70 • textbook lessons 34–35

Use the worked and guided tasks selectively. The eight core paper tasks are prerequisites, independent tasks and exits; further practice and extensions are optional.

Ordinary route: prerequisites 4, short inquiry 5, worked/guided 8, independent 15, correction 5, exit 3 minutes. Advanced route: inquiry 18, organise 4, selected practice 13, exit 5 minutes.

Prerequisites: Use substitution/elimination with two unknowns and check a solution in every original equation.

Ordinary route: use the first fixed case for the five-minute demonstration. Advanced route: complete the full investigation.

Inquiry and discussion: 18 minutes on paper: start with x+y+z=9, x−y=−1, y−z=−1. Predict the order of the three values (3 min). Two groups eliminate a different chosen unknown consistently, each producing a two-variable system and then the complete triple (7 min). Compare results and check all three original equations (4 min). Change only the first total from 9 to 12; predict and verify how every value changes (3 min). State why obtaining only two values does not complete a three-variable solution (1 min). No three-variable interactive engine is claimed.

Worked example 1

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (1)x + (1)y + (1)z = 9 \\ (1)x + (-1)y + (0)z = -1 \\ (0)x + (1)y + (-1)z = -1 \end{cases}$$

Reference answer

$x = 2, \quad y = 3, \quad z = 4.$

$$x = 2, \quad y = 3, \quad z = 4$$

1. Equation 2 minus 1 times equation 1 gives $(0)x + (-2)y + (-1)z = -10$.
2. Equation 3 minus $\frac{1}{2}$ times equation 2 gives $(0)x + (0)y + (\frac{-3}{2})z = -6$.
3. Back-substitute to obtain $z = 4$.
4. Back-substitute to obtain $y = 3$.
5. Back-substitute to obtain $x = 2$.
6. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Guided attempt 2

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (1)x + (1)y + (1)z = 6 \\ (1)x + (-1)y + (0)z = 0 \\ (0)x + (1)y + (-1)z = 0 \end{cases}$$

1. From $x - y = 0$, write $y = \square$.
2. From $y - z = 0$, write $z = y = \square$.
3. Substitute in the first equation: $x + x + x = 6$, so $x = \square$; recover y,z and check all three.

Reference answer

$x = 2, \quad y = 2, \quad z = 2.$

$$x = 2, \quad y = 2, \quad z = 2$$

1. Equation 2 minus 1 times equation 1 gives $(0)x + (-2)y + (-1)z = -6$.
2. Equation 3 minus $\frac{1}{2}$ times equation 2 gives $(0)x + (0)y + (\frac{-3}{2})z = -3$.
3. Back-substitute to obtain $z = 2$.
4. Back-substitute to obtain $y = 2$.
5. Back-substitute to obtain $x = 2$.
6. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Prerequisite 3

Does $x = 1, y = 2, z = 3$ solve the displayed system? Check every equation.

$$\begin{cases} (1)x + (1)y + (1)z = 6 \\ (1)x + (-1)y + (0)z = -1 \\ (0)x + (1)y + (-1)z = -1 \end{cases}$$

A. Yes, every original equation holds.

B. No, checking one equation is enough to reject it.

Reference answer

Yes, every original equation holds.

1. Equation 1 gives left side 6 and right side 6.
2. Equation 2 gives left side -1 and right side -1.
3. Equation 3 gives left side -1 and right side -1.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Prerequisite 4

To reduce three equations in x,y,z to a two-variable system, what should the two derived equations have in common?

A. They both eliminate the same unknown and retain the same other two.

B. One eliminates x and the other only y, leaving three unknowns together.

C. Their right sides must both be zero.

Reference answer

They both eliminate the same unknown and retain the same other two.

1. The reduced pair must involve just two common unknowns.
2. Otherwise it is not yet a two-variable system.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 5

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (1)x + (1)y + (1)z = 12 \\ (1)x + (-1)y + (0)z = 2 \\ (0)x + (1)y + (-1)z = -1 \end{cases}$$

Reference answer

$x = 5, \quad y = 3, \quad z = 4.$

$$x = 5, \quad y = 3, \quad z = 4$$

1. Equation 2 minus 1 times equation 1 gives $(0)x + (-2)y + (-1)z = -10$.
2. Equation 3 minus $\frac{1}{2}$ times equation 2 gives $(0)x + (0)y + (\frac{-3}{2})z = -6$.
3. Back-substitute to obtain $z = 4$.
4. Back-substitute to obtain $y = 3$.
5. Back-substitute to obtain $x = 5$.
6. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 6

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (1)x + (1)y + (1)z = 10 \\ (2)x + (-1)y + (0)z = -1 \\ (0)x + (1)y + (-1)z = 2 \end{cases}$$

Reference answer

$x = 2, \quad y = 5, \quad z = 3.$

$$x = 2, \quad y = 5, \quad z = 3$$

1. Equation 2 minus 2 times equation 1 gives $(0)x + (-3)y + (-2)z = -21$.
2. Equation 3 minus $\frac{-1}{3}$ times equation 2 gives $(0)x + (0)y + (\frac{-5}{3})z = -5$.
3. Back-substitute to obtain $z = 3$.
4. Back-substitute to obtain $y = 5$.
5. Back-substitute to obtain $x = 2$.
6. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 7

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (1)x + (1)y + (0)z = 5 \\ (0)x + (1)y + (1)z = 7 \\ (1)x + (0)y + (1)z = 6 \end{cases}$$

Reference answer

$x = 2, \quad y = 3, \quad z = 4.$

$$x = 2, \quad y = 3, \quad z = 4$$

1. Equation 3 minus 1 times equation 1 gives $(0)x + (-1)y + (1)z = 1$.
2. Equation 3 minus -1 times equation 2 gives $(0)x + (0)y + (2)z = 8$.
3. Back-substitute to obtain $z = 4$.
4. Back-substitute to obtain $y = 3$.
5. Back-substitute to obtain $x = 2$.
6. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 8

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (1)x + (1)y + (1)z = 3 \\ (1)x + (-1)y + (0)z = 1 \\ (0)x + (1)y + (-1)z = 1 \end{cases}$$

Reference answer

$x = 2, \quad y = 1, \quad z = 0.$

$$x = 2, \quad y = 1, \quad z = 0$$

1. Equation 2 minus 1 times equation 1 gives $(0)x + (-2)y + (-1)z = -2$.
2. Equation 3 minus $\frac{-1}{2}$ times equation 2 gives $(0)x + (0)y + (\frac{-3}{2})z = 0$.
3. Back-substitute to obtain $z = 0$.
4. Back-substitute to obtain $y = 1$.
5. Back-substitute to obtain $x = 2$.
6. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Exit question 9

Does $x = 2, y = 3, z = 4$ solve the displayed system? Check every equation.

$$\begin{cases} (1)x + (1)y + (1)z = 9 \\ (1)x + (-1)y + (0)z = -1 \\ (0)x + (1)y + (-1)z = -1 \end{cases}$$

A. Yes, every original equation holds.

B. No, checking one equation is enough to reject it.

Reference answer

Yes, every original equation holds.

1. Equation 1 gives left side 9 and right side 9.
2. Equation 2 gives left side -1 and right side -1.
3. Equation 3 gives left side -1 and right side -1.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Exit question 10

After elimination gives y=3 and z=4, the original first equation is x+y+z=9. What remains to finish the solution?

A. Find x=2, then check all three original equations.

B. Stop; two values are enough.

C. Choose any x without checking.

Reference answer

Find x=2, then check all three original equations.

1. Substitute 3 and 4 into x+y+z=9.
2. x=9−3−4=2; verify the complete triple.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 11

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (1)x + (1)y + (1)z = 7 \\ (1)x + (-1)y + (0)z = 2 \\ (0)x + (1)y + (-1)z = 1 \end{cases}$$

Reference answer

$x = 4, \quad y = 2, \quad z = 1.$

$$x = 4, \quad y = 2, \quad z = 1$$

1. Equation 2 minus 1 times equation 1 gives $(0)x + (-2)y + (-1)z = -5$.
2. Equation 3 minus $\frac{-1}{2}$ times equation 2 gives $(0)x + (0)y + (\frac{-3}{2})z = \frac{-3}{2}$.
3. Back-substitute to obtain $z = 1$.
4. Back-substitute to obtain $y = 2$.
5. Back-substitute to obtain $x = 4$.
6. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 12

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (1)x + (1)y + (1)z = 3 \\ (2)x + (-1)y + (0)z = 0 \\ (0)x + (1)y + (-1)z = \frac{-1}{2} \end{cases}$$

Reference answer

$x = \frac{1}{2}, \quad y = 1, \quad z = \frac{3}{2}.$

$$x = \frac{1}{2}, \quad y = 1, \quad z = \frac{3}{2}$$

1. Equation 2 minus 2 times equation 1 gives $(0)x + (-3)y + (-2)z = -6$.
2. Equation 3 minus $\frac{-1}{3}$ times equation 2 gives $(0)x + (0)y + (\frac{-5}{3})z = \frac{-5}{2}$.
3. Back-substitute to obtain $z = \frac{3}{2}$.
4. Back-substitute to obtain $y = 1$.
5. Back-substitute to obtain $x = \frac{1}{2}$.
6. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 13

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (2)x + (1)y + (-1)z = 1 \\ (1)x + (-1)y + (1)z = 2 \\ (1)x + (1)y + (1)z = 6 \end{cases}$$

Reference answer

$x = 1, \quad y = 2, \quad z = 3.$

$$x = 1, \quad y = 2, \quad z = 3$$

1. Equation 2 minus $\frac{1}{2}$ times equation 1 gives $(0)x + (\frac{-3}{2})y + (\frac{3}{2})z = \frac{3}{2}$.
2. Equation 3 minus $\frac{1}{2}$ times equation 1 gives $(0)x + (\frac{1}{2})y + (\frac{3}{2})z = \frac{11}{2}$.
3. Equation 3 minus $\frac{-1}{3}$ times equation 2 gives $(0)x + (0)y + (2)z = 6$.
4. Back-substitute to obtain $z = 3$.
5. Back-substitute to obtain $y = 2$.
6. Back-substitute to obtain $x = 1$.
7. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 14

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (1)x + (-2)y + (0)z = 0 \\ (0)x + (1)y + (-2)z = 0 \\ (1)x + (1)y + (1)z = 21 \end{cases}$$

Reference answer

$x = 12, \quad y = 6, \quad z = 3.$

$$x = 12, \quad y = 6, \quad z = 3$$

1. Equation 3 minus 1 times equation 1 gives $(0)x + (3)y + (1)z = 21$.
2. Equation 3 minus 3 times equation 2 gives $(0)x + (0)y + (7)z = 21$.
3. Back-substitute to obtain $z = 3$.
4. Back-substitute to obtain $y = 6$.
5. Back-substitute to obtain $x = 12$.
6. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 15

Three ticket types cost 2,3,5 dollars. Their counts are x,y,z. Nine tickets cost 33 dollars, and the first two types total 5 tickets. Form and solve a three-variable system; check integer counts.

Reference answer

$x = 2, \quad y = 3, \quad z = 4.$

$$x = 2, \quad y = 3, \quad z = 4$$

1. Counts give $x + y + z = 9$, revenue gives $2x + 3y + 5z = 33$, and $x + y = 5$.
2. Equation 2 minus 2 times equation 1 gives $(0)x + (1)y + (3)z = 15$.
3. Equation 3 minus 1 times equation 1 gives $(0)x + (0)y + (-1)z = -4$.
4. Back-substitute to obtain $z = 4$.
5. Back-substitute to obtain $y = 3$.
6. Back-substitute to obtain $x = 2$.
7. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 16

Do three written equations always determine three unique values? Describe the real solutions of this system.

$$\begin{cases} (1)x + (1)y + (1)z = 6 \\ (2)x + (2)y + (2)z = 12 \\ (1)x + (-1)y + (0)z = 0 \end{cases}$$

Reference answer

$x = t,\ y = t,\ z = 6 - 2t,\ t \in \mathbb{R}$; infinitely many.

$$x = t,\ y = t,\ z = 6 - 2t,\ t \in \mathbb{R}$$

1. The second equation is twice the first. The third says x=y.
2. Put x=y=t; then z=6−2t and every original equation holds.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.