Two-variable systems: quantities and models — Teacher answers

Year 7, lower volume • text pp. 61–66 • textbook lessons 31–33

Use the worked and guided tasks selectively. The eight core paper tasks are prerequisites, independent tasks and exits; further practice and extensions are optional.

Ordinary route: prerequisites 4, short inquiry 5, worked/guided 8, independent 15, correction 5, exit 3 minutes. Advanced route: inquiry 18, organise 4, selected practice 13, exit 5 minutes.

Prerequisites: Solve a two-variable system and distinguish counts, unit prices, totals, speeds and consistent time units.

Ordinary route: use the first fixed case for the five-minute demonstration. Advanced route: complete the full investigation.

Inquiry and discussion: 18 minutes using tickets: first set prices 2 and 5, total 10, revenue 32; name both unknown counts before revealing equations (3 min). Enumerate and solve, checking nonnegative integer counts and revenue (5 min). Change both ticket prices to the same value 3 while holding total 10: compare revenues 30 and 31 (6 min). Explain “insufficient independent information” versus “contradictory information”, and separate real solutions from feasible ticket counts (4 min). Same-price controls are supported by this existing lab.

Worked example 1

A notebook costs x dollars and a pen y dollars. 2 notebooks and 3 pens cost 16 dollars; 4 notebooks and 1 pens cost 12 dollars. Find both unit prices.

Reference answer

$x = 2, \quad y = 4.$

$$x = 2, \quad y = 4$$

1. With the stated variable meanings, the two independent conditions give $\begin{cases} (2)x + (3)y = 16 \\ (4)x + (1)y = 12 \end{cases}$.
2. Equation 2 minus 2 times equation 1 gives $(0)x + (-5)y = -20$.
3. Back-substitute to obtain $y = 4$.
4. Back-substitute to obtain $x = 2$.
5. Substitute the complete tuple into every original equation; each left side equals its stated right side.
6. Interpret both values in the stated units and check any nonnegative or integer restrictions.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Guided attempt 2

A tickets cost 2 dollars each and B tickets 5 dollars each. 10 tickets cost 32 dollars. Let x and y be the numbers of A and B tickets. Form and solve a system; check whole nonnegative counts.

1. Count condition: $x + y = \square$; revenue: $2x + 5y = \square$.
2. Use $x = 10 - y$: $2(10 - y) + 5y = 32$.
3. Then $3y = 32 - 20 = \square$; find y=____ and x=____, both whole nonnegative counts.

Reference answer

$x = 6, \quad y = 4.$

$$x = 6, \quad y = 4$$

1. With the stated variable meanings, the two independent conditions give $\begin{cases} (1)x + (1)y = 10 \\ (2)x + (5)y = 32 \end{cases}$.
2. Equation 2 minus 2 times equation 1 gives $(0)x + (3)y = 12$.
3. Back-substitute to obtain $y = 4$.
4. Back-substitute to obtain $x = 6$.
5. Substitute the complete tuple into every original equation; each left side equals its stated right side.
6. Interpret both values in the stated units and check any nonnegative or integer restrictions.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Prerequisite 3

x adults and y children visit a museum. Write their total number. Do not add ticket prices to a count.

Reference answer

$x + y$ people.

$$x + y$$

1. Both variables count people, so their units match.
2. The total is x+y.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Prerequisite 4

Convert 1 hour 30 minutes to hours before using a speed in km/h.

Reference answer

$\frac{3}{2}$ h.

$$1 + 30/60$$

1. 30 minutes is 30/60 hour.
2. The total is 3/2 hours.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 5

A tickets cost 3 dollars each and B tickets 5 dollars each. 12 tickets cost 44 dollars. Let x and y be the numbers of A and B tickets. Form and solve a system; check whole nonnegative counts.

Reference answer

$x = 8, \quad y = 4.$

$$x = 8, \quad y = 4$$

1. With the stated variable meanings, the two independent conditions give $\begin{cases} (1)x + (1)y = 12 \\ (3)x + (5)y = 44 \end{cases}$.
2. Equation 2 minus 3 times equation 1 gives $(0)x + (2)y = 8$.
3. Back-substitute to obtain $y = 4$.
4. Back-substitute to obtain $x = 8$.
5. Substitute the complete tuple into every original equation; each left side equals its stated right side.
6. Interpret both values in the stated units and check any nonnegative or integer restrictions.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 6

A notebook costs x dollars and a pen y dollars. 3 notebooks and 2 pens cost 19 dollars; 2 notebooks and 5 pens cost 31 dollars. Find both unit prices.

Reference answer

$x = 3, \quad y = 5.$

$$x = 3, \quad y = 5$$

1. With the stated variable meanings, the two independent conditions give $\begin{cases} (3)x + (2)y = 19 \\ (2)x + (5)y = 31 \end{cases}$.
2. Equation 2 minus $\frac{2}{3}$ times equation 1 gives $(0)x + (\frac{11}{3})y = \frac{55}{3}$.
3. Back-substitute to obtain $y = 5$.
4. Back-substitute to obtain $x = 3$.
5. Substitute the complete tuple into every original equation; each left side equals its stated right side.
6. Interpret both values in the stated units and check any nonnegative or integer restrictions.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 7

A boat travels downstream at 28 km/h and upstream at 20 km/h. Let x be its still-water speed and y the current speed, both in km/h. Find x and y.

Reference answer

$x = 24, \quad y = 4.$

$$x = 24, \quad y = 4$$

1. With the stated variable meanings, the two independent conditions give $\begin{cases} (1)x + (1)y = 28 \\ (1)x + (-1)y = 20 \end{cases}$.
2. Equation 2 minus 1 times equation 1 gives $(0)x + (-2)y = -8$.
3. Back-substitute to obtain $y = 4$.
4. Back-substitute to obtain $x = 24$.
5. Substitute the complete tuple into every original equation; each left side equals its stated right side.
6. Interpret both values in the stated units and check any nonnegative or integer restrictions.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 8

A group has 18 people. Adults number twice the children. Let x and y be adult and child counts. Find both counts.

Reference answer

$x = 12, \quad y = 6.$

$$x = 12, \quad y = 6$$

1. With the stated variable meanings, the two independent conditions give $\begin{cases} (1)x + (-2)y = 0 \\ (1)x + (1)y = 18 \end{cases}$.
2. Equation 2 minus 1 times equation 1 gives $(0)x + (3)y = 18$.
3. Back-substitute to obtain $y = 6$.
4. Back-substitute to obtain $x = 12$.
5. Substitute the complete tuple into every original equation; each left side equals its stated right side.
6. Interpret both values in the stated units and check any nonnegative or integer restrictions.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Exit question 9

If x is a ticket count and p is dollars per ticket, what is px?

A. The total count in tickets.

B. A speed.

C. The total price in dollars.

Reference answer

The total price in dollars.

1. Multiply dollars per ticket by ticket count.
2. Ticket units cancel and dollars remain.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Exit question 10

Solving a ticket model gives x=11/3, y=7/3. May these be rounded to 4 and 2 and reported as the exact solution? Explain.

Reference answer

No. Ticket counts must be integers and rounding can break the original revenue condition.

1. For 6 tickets priced 2 and 5 dollars, the fractional pair gives revenue 19.
2. The rounded pair (4,2) gives 18 dollars, so it is not a solution.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 11

A rectangle has perimeter 30 cm. Its length x is 3 cm greater than its width y. Find both dimensions.

Reference answer

$x = 9, \quad y = 6.$

$$x = 9, \quad y = 6$$

1. With the stated variable meanings, the two independent conditions give $\begin{cases} (1)x + (1)y = 15 \\ (1)x + (-1)y = 3 \end{cases}$.
2. Equation 2 minus 1 times equation 1 gives $(0)x + (-2)y = -12$.
3. Back-substitute to obtain $y = 6$.
4. Back-substitute to obtain $x = 9$.
5. Substitute the complete tuple into every original equation; each left side equals its stated right side.
6. Interpret both values in the stated units and check any nonnegative or integer restrictions.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 12

A notebook costs x dollars and a pen y dollars. 4 notebooks and 3 pens cost 27 dollars; 2 notebooks and 5 pens cost 31 dollars. Find both unit prices.

Reference answer

$x = 3, \quad y = 5.$

$$x = 3, \quad y = 5$$

1. With the stated variable meanings, the two independent conditions give $\begin{cases} (4)x + (3)y = 27 \\ (2)x + (5)y = 31 \end{cases}$.
2. Equation 2 minus $\frac{1}{2}$ times equation 1 gives $(0)x + (\frac{7}{2})y = \frac{35}{2}$.
3. Back-substitute to obtain $y = 5$.
4. Back-substitute to obtain $x = 3$.
5. Substitute the complete tuple into every original equation; each left side equals its stated right side.
6. Interpret both values in the stated units and check any nonnegative or integer restrictions.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 13

A boat travels downstream at 34 km/h and upstream at 26 km/h. Let x be its still-water speed and y the current speed, both in km/h. Find x and y.

Reference answer

$x = 30, \quad y = 4.$

$$x = 30, \quad y = 4$$

1. With the stated variable meanings, the two independent conditions give $\begin{cases} (1)x + (1)y = 34 \\ (1)x + (-1)y = 26 \end{cases}$.
2. Equation 2 minus 1 times equation 1 gives $(0)x + (-2)y = -8$.
3. Back-substitute to obtain $y = 4$.
4. Back-substitute to obtain $x = 30$.
5. Substitute the complete tuple into every original equation; each left side equals its stated right side.
6. Interpret both values in the stated units and check any nonnegative or integer restrictions.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 14

Machine A makes x items per hour and B y items per hour. Two hours of A plus one hour of B make 26 items; one hour of A plus three hours of B make 33 items. Find both hourly outputs.

Reference answer

$x = 9, \quad y = 8.$

$$x = 9, \quad y = 8$$

1. With the stated variable meanings, the two independent conditions give $\begin{cases} (2)x + (1)y = 26 \\ (1)x + (3)y = 33 \end{cases}$.
2. Equation 2 minus $\frac{1}{2}$ times equation 1 gives $(0)x + (\frac{5}{2})y = 20$.
3. Back-substitute to obtain $y = 8$.
4. Back-substitute to obtain $x = 9$.
5. Substitute the complete tuple into every original equation; each left side equals its stated right side.
6. Interpret both values in the stated units and check any nonnegative or integer restrictions.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 15

Two crops occupy x and y hectares, totalling 100 hectares. Their yields are 2 and 3 tonnes per hectare, with total yield 260 tonnes. Find each crop area, then check both units.

Reference answer

$x = 40, \quad y = 60.$

$$x = 40, \quad y = 60$$

1. With the stated variable meanings, the two independent conditions give $\begin{cases} (1)x + (1)y = 100 \\ (2)x + (3)y = 260 \end{cases}$.
2. Equation 2 minus 2 times equation 1 gives $(0)x + (1)y = 60$.
3. Back-substitute to obtain $y = 60$.
4. Back-substitute to obtain $x = 40$.
5. Substitute the complete tuple into every original equation; each left side equals its stated right side.
6. Interpret both values in the stated units and check any nonnegative or integer restrictions.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 16

A tickets cost 2 dollars each and B tickets 5 dollars each. 10 tickets cost 60 dollars. Let x and y be the numbers of A and B tickets. Form and solve a system; check whole nonnegative counts.

Reference answer

No feasible ticket counts. The real solution is $x = -\frac{10}{3}$, $y = \frac{40}{3}$, and 60 dollars exceeds the maximum possible 50.

$$x = \frac{-10}{3}, \quad y = \frac{40}{3}$$

1. With the stated variable meanings, the two independent conditions give $\begin{cases} (1)x + (1)y = 10 \\ (2)x + (5)y = 60 \end{cases}$.
2. Equation 2 minus 2 times equation 1 gives $(0)x + (3)y = 40$.
3. Back-substitute to obtain $y = \frac{40}{3}$.
4. Back-substitute to obtain $x = \frac{-10}{3}$.
5. Substitute the complete tuple into every original equation; each left side equals its stated right side.
6. Interpret both values in the stated units and check any nonnegative or integer restrictions.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.