Two-variable systems: substitution and elimination — Teacher answers

Year 7, lower volume • text pp. 51–60 • textbook lessons 26–30

Use the worked and guided tasks selectively. The eight core paper tasks are prerequisites, independent tasks and exits; further practice and extensions are optional.

Ordinary route: prerequisites 4, short inquiry 5, worked/guided 8, independent 15, correction 5, exit 3 minutes. Advanced route: inquiry 18, organise 4, selected practice 13, exit 5 minutes.

Prerequisites: Solve one-variable equations, distribute a multiplier and substitute a signed value into an expression.

Ordinary route: use the first fixed case for the five-minute demonstration. Advanced route: complete the full investigation.

Inquiry and discussion: 18 minutes using tickets: A=2 dollars, B=5 dollars, total 10, revenue 32. Predict a feasible pair (2 min), list possible integer pairs and the two equations (4 min), then have two groups solve the same model by substitution and by scaled subtraction (7 min). Compare how each method removes one unknown and check both original conditions (3 min). Change only revenue to 31 and explain why the real solution is not a feasible whole-ticket solution (2 min); do not round counts.

Worked example 1

Use substitution to solve the system, then check both equations.

$$\begin{cases} (1)x + (1)y = 7 \\ (-2)x + (1)y = 1 \end{cases}$$

Reference answer

$x = 2, \quad y = 5.$

$$x = 2, \quad y = 5$$

1. The second equation gives $y = 2x + 1$.
2. Substitute in the first: $x + (2x + 1) = 7$, so $x = 2$.
3. Then $y = 5$. Check $2 + 5 = 7$ and $-2(2) + 5 = 1$.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Guided attempt 2

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (2)x + (1)y = 11 \\ (1)x + (-1)y = 1 \end{cases}$$

1. Add the original equations: $(2x + y) + (x - y) = 11 + 1$.
2. Collect: $3x = \square$, so $x = \square$.
3. Back-substitute: $y = 11 - 2(\square) = \square$; check both original equations.

Reference answer

$x = 4, \quad y = 3.$

$$x = 4, \quad y = 3$$

1. Equation 2 minus $\frac{1}{2}$ times equation 1 gives $(0)x + (\frac{-3}{2})y = \frac{-9}{2}$.
2. Back-substitute to obtain $y = 3$.
3. Back-substitute to obtain $x = 4$.
4. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Prerequisite 3

Does $x = 3, y = 4$ solve the displayed system? Check every equation.

$$\begin{cases} (1)x + (1)y = 7 \\ (2)x + (-1)y = 2 \end{cases}$$

A. Yes, every original equation holds.

B. No, checking one equation is enough to reject it.

Reference answer

Yes, every original equation holds.

1. Equation 1 gives left side 7 and right side 7.
2. Equation 2 gives left side 2 and right side 2.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Prerequisite 4

Does $x = 2, y = 5$ solve the displayed system? Check every equation.

$$\begin{cases} (1)x + (1)y = 7 \\ (2)x + (-1)y = 2 \end{cases}$$

A. Yes, one matching equation is sufficient.

B. No, at least one original equation fails.

Reference answer

No, at least one original equation fails.

1. Equation 1 gives left side 7 and right side 7.
2. Equation 2 gives left side -1 and right side 2.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 5

Solve by substitution and check both equations.

$$\begin{cases} (1)x + (-1)y = 2 \\ (2)x + (1)y = 10 \end{cases}$$

Reference answer

$x = 4, \quad y = 2.$

$$x = 4, \quad y = 2$$

1. From the first equation, $x = y + 2$.
2. Substitute: $2(y + 2) + y = 10$, so $y = 2$.
3. Then $x = 4$. Both equations give their original right sides.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 6

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (2)x + (3)y = 16 \\ (3)x + (-2)y = 11 \end{cases}$$

Reference answer

$x = 5, \quad y = 2.$

$$x = 5, \quad y = 2$$

1. Equation 2 minus $\frac{3}{2}$ times equation 1 gives $(0)x + (\frac{-13}{2})y = -13$.
2. Back-substitute to obtain $y = 2$.
3. Back-substitute to obtain $x = 5$.
4. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 7

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (3)x + (-2)y = 0 \\ (2)x + (1)y = 7 \end{cases}$$

Reference answer

$x = 2, \quad y = 3.$

$$x = 2, \quad y = 3$$

1. Equation 2 minus $\frac{2}{3}$ times equation 1 gives $(0)x + (\frac{7}{3})y = 7$.
2. Back-substitute to obtain $y = 3$.
3. Back-substitute to obtain $x = 2$.
4. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 8

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (1)x + (1)y = 3 \\ (2)x + (-1)y = 1 \end{cases}$$

Reference answer

$x = \frac{4}{3}, \quad y = \frac{5}{3}.$

$$x = \frac{4}{3}, \quad y = \frac{5}{3}$$

1. Equation 2 minus 2 times equation 1 gives $(0)x + (-3)y = -5$.
2. Back-substitute to obtain $y = \frac{5}{3}$.
3. Back-substitute to obtain $x = \frac{4}{3}$.
4. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Exit question 9

Does $x = 2, y = 3$ solve the displayed system? Check every equation.

$$\begin{cases} (1)x + (2)y = 8 \\ (3)x + (-1)y = 3 \end{cases}$$

A. Yes, every original equation holds.

B. No, checking one equation is enough to reject it.

Reference answer

Yes, every original equation holds.

1. Equation 1 gives left side 8 and right side 8.
2. Equation 2 gives left side 3 and right side 3.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Exit question 10

To multiply $2x - y = 5$ by 3, which equation is correct?

A. $6x - 3y = 15$

B. $6x - y = 15$

C. $6x - 3y = 5$

Reference answer

$6x - 3y = 15$

1. Multiply each term on both sides by 3.
2. The coefficients and right side all scale.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 11

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (1)x + (1)y = 9 \\ (1)x + (-1)y = 3 \end{cases}$$

Reference answer

$x = 6, \quad y = 3.$

$$x = 6, \quad y = 3$$

1. Equation 2 minus 1 times equation 1 gives $(0)x + (-2)y = -6$.
2. Back-substitute to obtain $y = 3$.
3. Back-substitute to obtain $x = 6$.
4. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 12

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (2)x + (3)y = 13 \\ (2)x + (-1)y = 1 \end{cases}$$

Reference answer

$x = 2, \quad y = 3.$

$$x = 2, \quad y = 3$$

1. Equation 2 minus 1 times equation 1 gives $(0)x + (-4)y = -12$.
2. Back-substitute to obtain $y = 3$.
3. Back-substitute to obtain $x = 2$.
4. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 13

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (3)x + (2)y = 19 \\ (5)x + (-3)y = 0 \end{cases}$$

Reference answer

$x = 3, \quad y = 5.$

$$x = 3, \quad y = 5$$

1. Equation 2 minus $\frac{5}{3}$ times equation 1 gives $(0)x + (\frac{-19}{3})y = \frac{-95}{3}$.
2. Back-substitute to obtain $y = 5$.
3. Back-substitute to obtain $x = 3$.
4. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 14

Solve the system. Show elimination and check every original equation.

$$\begin{cases} (1)x + (1)y = 1 \\ (2)x + (-1)y = -7 \end{cases}$$

Reference answer

$x = -2, \quad y = 3.$

$$x = -2, \quad y = 3$$

1. Equation 2 minus 2 times equation 1 gives $(0)x + (-3)y = -9$.
2. Back-substitute to obtain $y = 3$.
3. Back-substitute to obtain $x = -2$.
4. Substitute the complete tuple into every original equation; each left side equals its stated right side.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 15

Describe every real solution of the system. Explain why two written equations do not guarantee a unique pair.

$$\begin{cases} (1)x + (1)y = 6 \\ (2)x + (2)y = 12 \end{cases}$$

Reference answer

$(x, y) = (t, 6 - t),\ t \in \mathbb{R}$; infinitely many solutions.

$$(x, y) = (t, 6 - t),\ t \in \mathbb{R}$$

1. The second equation is twice the first, so it adds no new restriction.
2. Choose any real x=t and set y=6−t; both equations hold.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 16

Decide whether the system has a real solution. Explain by elimination.

$$\begin{cases} (1)x + (1)y = 6 \\ (2)x + (2)y = 13 \end{cases}$$

Reference answer

No solution. Subtracting twice the first from the second gives 0=1.

1. Twice the first equation is 2x+2y=12.
2. The second requires the same left side to equal 13, a contradiction.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.