Square roots and cube roots: meanings and signs — Teacher answers

Year 7, lower volume • text pp. 29–34 • textbook lessons 15–17

Use the worked and guided tasks selectively. The eight core paper tasks are prerequisites, independent tasks and exits; further practice and extensions are optional.

Ordinary route: prerequisites 4, short inquiry 5, worked/guided 8, independent 15, correction 5, exit 3 minutes. Advanced route: inquiry 18, organise 4, selected practice 13, exit 5 minutes.

Prerequisites: Calculate squares and cubes, including signed numbers and simple fractions.

Ordinary route: use the first fixed case for the five-minute demonstration. Advanced route: complete the full investigation.

Inquiry and discussion: 18 minutes: in the roots lab compare square areas 9, 2 and 0, predicting the allowed physical side lengths before checking (3 min prediction, 6 min trials). For area 9, contrast side length 3 with both solutions of $x^2=9$; for area 0 explain why there is one value (4 min). On paper change the equation to $x^3=-8$: find a negative cube root, and explain why this is not a negative square side (3 min). Sort the three requests “square roots”, “principal square root”, “cube root” and state the sign rules (2 min). The lab itself models only nonnegative square roots.

Worked example 1

Find all square roots of 81.

Reference answer

± 9.

$$\pm 9$$

1. For a positive radicand, both opposite values have the required square.
2. Check $(9)^2 = 81$. The negative of this nonzero root has the same square.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Guided attempt 2

Evaluate $\sqrt[3]{-64}$.

1. Find r with $r^3 = -64$; its sign is ____.
2. Complete $(-\square)^3 = -64$.
3. Therefore $\sqrt[3]{-64} = \square$; cube it to check.

Reference answer

-4.

$$-4$$

1. A cube root cubes to the given number.
2. Check $(-4)^3 = -64$.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Prerequisite 3

Evaluate $\sqrt{49}$.

Reference answer

7.

$$7$$

1. The radical symbol denotes the nonnegative square root.
2. Check $(7)^2 = 49$.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Prerequisite 4

Evaluate $\sqrt{0}$.

Reference answer

0.

$$0$$

1. The radical symbol denotes the nonnegative square root.
2. Check $(0)^2 = 0$.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 5

Evaluate $\sqrt{\frac{25}{36}}$.

Reference answer

$\frac{5}{6}$.

$$\frac{5}{6}$$

1. The radical symbol denotes the nonnegative square root.
2. Check $(\frac{5}{6})^2 = \frac{25}{36}$.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 6

Find all square roots of $\frac{9}{100}$.

Reference answer

$\pm\frac{3}{10}$.

$$\pm\frac{3}{10}$$

1. For a positive radicand, both opposite values have the required square.
2. Check $(\frac{3}{10})^2 = \frac{9}{100}$. The negative of this nonzero root has the same square.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 7

Evaluate $\sqrt[3]{\frac{-27}{125}}$.

Reference answer

$\frac{-3}{5}$.

$$\frac{-3}{5}$$

1. A cube root cubes to the given number.
2. Check $(\frac{-3}{5})^3 = \frac{-27}{125}$.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 8

Evaluate $\sqrt{(-8)^2}$. Explain why the result is not −8.

Reference answer

8; a principal square root is nonnegative.

$$8$$

1. First $(-8)^2=64$.
2. The nonnegative square root of 64 is 8; in general $\sqrt{(a^2)}=|a|$.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Exit question 9

Does −9 have a real square root?

A. Yes, −3.

B. Yes, both 3 and −3.

C. No; a real square cannot be negative.

Reference answer

No; a real square cannot be negative.

1. Both 3^2 and $(-3)^2$ are positive 9.
2. No real number has square −9.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Exit question 10

Evaluate $\sqrt[3]{-8}$.

Reference answer

-2.

$$-2$$

1. A cube root cubes to the given number.
2. Check $(-2)^3 = -8$.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 11

Evaluate $\sqrt{100}$.

Reference answer

10.

$$10$$

1. The radical symbol denotes the nonnegative square root.
2. Check $(10)^2 = 100$.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 12

Find all square roots of $\frac{4}{49}$.

Reference answer

$\pm\frac{2}{7}$.

$$\pm\frac{2}{7}$$

1. For a positive radicand, both opposite values have the required square.
2. Check $(\frac{2}{7})^2 = \frac{4}{49}$. The negative of this nonzero root has the same square.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 13

Evaluate $\sqrt[3]{216}$.

Reference answer

6.

$$6$$

1. A cube root cubes to the given number.
2. Check $(6)^3 = 216$.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 14

Evaluate $\sqrt[3]{\frac{1}{64}}$.

Reference answer

$\frac{1}{4}$.

$$\frac{1}{4}$$

1. A cube root cubes to the given number.
2. Check $(\frac{1}{4})^3 = \frac{1}{64}$.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 15

For which real x is $\sqrt{x-3}$ defined? Include the endpoint.

Reference answer

$x \geq 3$.

$$x \geq 3$$

1. A real square-root radicand must be nonnegative: x−3≥0.
2. Thus x≥3; equality gives $\sqrt{0}$=0.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 16

Solve $x^2 = 12$ exactly. Explain how this differs from writing $\sqrt{12}$.

Reference answer

$x = \pm\sqrt{12}$; the radical alone is the nonnegative root.

$$x = \pm\sqrt{12}$$

1. Both the positive and negative square roots square to 12.
2. The equation has two solutions; $\sqrt{12}$ names just one of them.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.