Real numbers: classification, bounds and approximation — Teacher answers

Year 7, lower volume • text pp. 35–38 • textbook lessons 18–19

Use the worked and guided tasks selectively. The eight core paper tasks are prerequisites, independent tasks and exits; further practice and extensions are optional.

Ordinary route: prerequisites 4, short inquiry 5, worked/guided 8, independent 15, correction 5, exit 3 minutes. Advanced route: inquiry 18, organise 4, selected practice 13, exit 5 minutes.

Prerequisites: Distinguish principal square roots from two square roots; compare nonnegative squares and round decimals.

Ordinary route: use the first fixed case for the five-minute demonstration. Advanced route: complete the full investigation.

Inquiry and discussion: 18 minutes in the roots lab with target area 2: predict whether 1.4 is exact (3 min). Hold the target fixed while changing precision from 0 to 1 to 2 decimal places; square trial sides 1.4,1.41,1.42 and record exact lower/upper comparisons (8 min). Change only the target to 9 and explain why its bounds collapse to one exact value (3 min). Write a strict bound for $\sqrt{2}$ and explain why neither a narrow interval nor a finite display proves the entire decimal expansion or irrationality (4 min).

Worked example 1

Use $\frac{7}{5}$ and $\frac{3}{2}$ to give strict bounds for $\sqrt{2}$. Justify by squaring.

Reference answer

$\frac{7}{5} < \sqrt{2} < \frac{3}{2}$.

$$\frac{7}{5} < \sqrt{2} < \frac{3}{2}$$

1. Both bounds are nonnegative, where squaring preserves order.
2. Their squares are $\frac{49}{25}$ and $\frac{9}{4}$, with $\frac{49}{25} < 2 < \frac{9}{4}$.
3. Therefore the square root lies strictly between the two bounds.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Guided attempt 2

Given $\sqrt{5} \approx 2.2361$, estimate $4 - \sqrt{5}$ to two decimal places.

1. Substitute the supplied decimal: $4 - \sqrt{5} \approx 4 - 2.2361$.
2. Before rounding, $4 - 2.2361 = \square$.
3. The third decimal digit is ____; to two places the estimate is ____.

Reference answer

1.76.

$$4 - \sqrt{5} \approx 1.76$$

1. Use the supplied approximation: 4−2.2361=1.7639.
2. Round to two decimal places: approximately 1.76, not an exact equality.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Prerequisite 3

Classify $\sqrt{9}$.

A. Rational. B. Irrational.

Reference answer

Rational.

1. $\sqrt{9}$=3, an integer.
2. Use an exact value or the stated repeating/nonrepeating definition, not a finite calculator display.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Prerequisite 4

Classify 0.125.

A. Irrational.

B. Rational.

Reference answer

Rational.

1. 0.125=1/8, a ratio of integers.
2. Use an exact value or the stated repeating/nonrepeating definition, not a finite calculator display.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 5

Classify $0.\overline{12}$. The bar means the block repeats forever.

A. Rational.

B. Irrational.

Reference answer

Rational.

1. The block 12 repeats forever; its value is 12/99=4/33.
2. Use an exact value or the stated repeating/nonrepeating definition, not a finite calculator display.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 6

Compare $-\sqrt{5}$ and $-\sqrt{6}$. Explain the effect of the negative signs.

Reference answer

$-\sqrt{5} > -\sqrt{6}$.

$$-\sqrt{5} > -\sqrt{6}$$

1. 5<6 gives $\sqrt{5}<\sqrt{6}$ for nonnegative principal roots.
2. Taking opposites reverses order.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 7

Find $|-\sqrt{7}|$ and the opposite of $-\sqrt{7}$.

Reference answer

Both are $\sqrt{7}$.

$$|-\sqrt{7}| = \sqrt{7}$$

1. $-\sqrt{7}$ is negative; its distance from zero is $\sqrt{7}$.
2. Its opposite changes sign, also giving $\sqrt{7}$.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 8

Use $\frac{173}{100}$ and $\frac{174}{100}$ to give strict bounds for $\sqrt{3}$. Justify by squaring.

Reference answer

$\frac{173}{100} < \sqrt{3} < \frac{174}{100}$.

$$\frac{173}{100} < \sqrt{3} < \frac{174}{100}$$

1. Both bounds are nonnegative, where squaring preserves order.
2. Their squares are $\frac{29929}{10000}$ and $\frac{30276}{10000}$, with $\frac{29929}{10000} < 3 < \frac{30276}{10000}$.
3. Therefore the square root lies strictly between the two bounds.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Exit question 9

Which statement is justified by $1.41^2<2<1.42^2$?

A. $\sqrt{2} = 1.41$

B. These two calculations alone prove every digit of $\sqrt{2}$.

C. $1.41 < \sqrt{2} < 1.42$

Reference answer

$1.41 < \sqrt{2} < 1.42$

1. Both endpoints are positive and their squares bracket 2.
2. This proves a bound, not equality or an entire decimal expansion.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Exit question 10

Classify $\sqrt{2}$.

A. Rational.

B. Irrational.

Reference answer

Irrational.

1. 2 is not a perfect square; use the taught irrationality result for $\sqrt{2}$.
2. Use an exact value or the stated repeating/nonrepeating definition, not a finite calculator display.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 11

Classify $-\sqrt{16}$.

A. Rational.

B. Irrational.

Reference answer

Rational.

1. $-\sqrt{16}=-4$.
2. Use an exact value or the stated repeating/nonrepeating definition, not a finite calculator display.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 12

Classify $\sqrt[3]{27}$.

A. Irrational. B. Rational.

Reference answer

Rational.

1. The cube root of 27 is 3.
2. Use an exact value or the stated repeating/nonrepeating definition, not a finite calculator display.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 13

Simplify $3\sqrt{2} - \sqrt{2}$ exactly, without replacing $\sqrt{2}$ by a decimal.

$$3\sqrt{2} - \sqrt{2}$$

Reference answer

$2\sqrt{2}$.

$$2\sqrt{2}$$

1. Factor the common real number $\sqrt{2}$: $(3-1)\sqrt{2}$.
2. The exact result is $2\sqrt{2}$.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 14

Use 2 and 3 to give strict bounds for $\sqrt{5}$. Justify by squaring.

Reference answer

$2 < \sqrt{5} < 3$.

$$2 < \sqrt{5} < 3$$

1. Both bounds are nonnegative, where squaring preserves order.
2. Their squares are $\frac{4}{1}$ and $\frac{9}{1}$, with $\frac{4}{1} < 5 < \frac{9}{1}$.
3. Therefore the square root lies strictly between the two bounds.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 15

A screen shows 0.333333. Can that finite display alone decide whether the intended number is 1/3 or a terminating decimal? Explain.

Reference answer

No. Both 1/3 and 333333/1000000 can give that rounded display; the intended definition matters.

1. A rounded display omits information about later digits.
2. Use exact notation or a specified repeating rule before classifying.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 16

Use $\frac{223}{100}$ and $\frac{224}{100}$ to give strict bounds for $\sqrt{5}$. Justify by squaring.

Reference answer

$\frac{223}{100} < \sqrt{5} < \frac{224}{100}$.

$$\frac{223}{100} < \sqrt{5} < \frac{224}{100}$$

1. Both bounds are nonnegative, where squaring preserves order.
2. Their squares are $\frac{49729}{10000}$ and $\frac{50176}{10000}$, with $\frac{49729}{10000} < 5 < \frac{50176}{10000}$.
3. Therefore the square root lies strictly between the two bounds.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.