Parallel lines: conditions and reasoning — Teacher answers

Year 7, lower volume · text pp. 9–24 · textbook lessons 5–12

Use the worked and guided tasks selectively. The eight core paper tasks are prerequisites, independent tasks and exits; further practice and extensions are optional.

Ordinary route: prerequisites 4, short inquiry 5, worked/guided 8, independent 15, correction 5, exit 3 minutes. Advanced route: inquiry 18, organise 4, selected practice 13, exit 5 minutes.

Prerequisites: Know vertical/supplementary angles and identify corresponding, alternate and same-side interior positions.

Ordinary route: use the first fixed case for the five-minute demonstration. Advanced route: complete the full investigation.

Inquiry and discussion: 18 minutes using the existing parallels lab: predict which relations survive if parallelism is removed (3 min). Set α=60°, locked parallel, and record corresponding, alternate interior, same-side interior and vertical angles (4 min). Unlock and change only β to 75°, then back to 60°; keep α fixed (5 min). Identify which equalities or sums change and which vertical-angle fact does not (4 min). Give one justified converse argument, distinguishing a test for parallelism from a property that assumes it (2 min).

Worked example 1

Coplanar lines a and b are cut by a transversal. A pair of corresponding angles are both 64°. Prove that a is parallel to b.

Reference answer

The corresponding angles are equal, so a is parallel to b.

1. Both marked angles are given as 64°, so they are equal.
2. Use the corresponding-angle test for parallel lines, not the property of lines already known parallel.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Guided attempt 2

Two parallel coplanar lines are cut by a transversal. One angle is 68°. Find its same-side interior angle; state the reason.

1. Parallel lines give same-side interior sum $68 + x = \square^\circ$.
2. Complete $x = 180 - \square$.
3. Write $x = \square^\circ$ and name the parallel-line property.

Reference answer

$112\,^\circ$.

$$180 - 68$$

1. Use the stated parallel condition before applying this angle relation.
2. The required angle is $180 - 68 = 112$ degrees.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Prerequisite 3

In “If two lines are parallel, their corresponding angles are equal”, identify the hypothesis.

A. The corresponding angles are equal.
B. The angles are supplementary.
C. The two lines are parallel.

Reference answer

The two lines are parallel.

1. The hypothesis is the information after “if”.
2. Angle equality is the conclusion of this statement.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Prerequisite 4

One angle is 72°. Find its vertically opposite angle; state the reason.

Reference answer

72 °.

$$72$$

1. Use the angle relation stated in the question.
2. The required angle is $72 = 72$ degrees.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 5

Two parallel coplanar lines are cut by a transversal. One angle is 57°. Find its corresponding angle; state the reason.

Reference answer

57 °.

$$57$$

1. Use the stated parallel condition before applying this angle relation.
2. The required angle is $57 = 57$ degrees.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 6

Two parallel coplanar lines are cut by a transversal. One angle is 113°. Find its alternate interior angle; state the reason.

Reference answer

113 °.

$$113$$

1. Use the stated parallel condition before applying this angle relation.
2. The required angle is $113 = 113$ degrees.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 7

Two parallel coplanar lines are cut by a transversal. One angle is 124°. Find its same-side interior angle; state the reason.

Reference answer

56 °.

$$180 - 124$$

1. Use the stated parallel condition before applying this angle relation.
2. The required angle is $180 - 124 = 56$ degrees.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 8

Two coplanar lines are cut by a transversal. A pair of same-side interior angles are 72° and 108°. Write a short proof that the lines are parallel.

Reference answer

Their sum is 180°, so the lines are parallel by the same-side-interior-angle test.

1. 72°+108°=180°, so this pair is supplementary.
2. They have the specified same-side interior positions; apply the converse test.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Exit question 9

Without a parallel condition, which statement remains always valid for two intersecting straight lines?

A. All corresponding angles at two intersections are equal. B. Every pair of interior angles is supplementary.
C. Vertically opposite angles are equal.

Reference answer

Vertically opposite angles are equal.

1. Vertically opposite angles use one intersection only.
2. The other claims need additional conditions.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Exit question 10

Two parallel coplanar lines are cut by a transversal. One angle is 39°. Find its same-side interior angle; state the reason.

Reference answer

$141\ ^\circ$.

$$180 - 39$$

1. Use the stated parallel condition before applying this angle relation.
2. The required angle is $180 - 39 = 141$ degrees.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 11

Two parallel coplanar lines are cut by a transversal. One angle is 46°. Find its alternate interior angle; state the reason.

Reference answer

$46\ ^\circ$.

$$46$$

1. Use the stated parallel condition before applying this angle relation.
2. The required angle is $46 = 46$ degrees.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 12

Two parallel coplanar lines are cut by a transversal. One angle is 95°. Find its same-side interior angle; state the reason.

Reference answer

85 °.

$$180 - 95$$

1. Use the stated parallel condition before applying this angle relation.
2. The required angle is $180 - 95 = 85$ degrees.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 13

A pair of corresponding angles are 55° and 65°. Can these given angles justify parallelism? Explain.

Reference answer

No. They are unequal; in this configuration the lines are not parallel.

1. The test needs equality, not merely both angles being acute.
2. 55° differs from 65°, so parallelism would contradict the corresponding-angle property.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 14

In one plane, a and b are each perpendicular to the same line c. Explain why a and b are parallel.

Reference answer

Corresponding right angles are equal; therefore a is parallel to b.

1. Both relevant angles are 90°.
2. Their corresponding positions give the parallel-line test.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 15

A learner writes “a is parallel to b because corresponding angles are equal; these angles are equal because a is parallel to b.” No equality was given. Diagnose and repair the reasoning.

Reference answer

This is circular. Supply an independent given angle equality or another valid parallel test before claiming parallelism.

1. Neither step supplies independent evidence.
2. For example, two given corresponding angles of 60° establish equality first, then parallelism.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 16

The true statement “vertically opposite angles are equal” does not imply that every pair of equal angles are vertically opposite. Give a counterexample.

Reference answer

No. Two separate right angles are equal but need not share a vertex.

1. A converse needs its own justification.
2. Right angles at (0,0) and (3,0) are both 90° but are not vertically opposite.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.