Inequality systems: overlap and feasible choices — Teacher answers

Year 7, lower volume • text pp. 79–88 • textbook lessons 40–44

Use the worked and guided tasks selectively. The eight core paper tasks are prerequisites, independent tasks and exits; further practice and extensions are optional.

Ordinary route: prerequisites 4, short inquiry 5, worked/guided 8, independent 15, correction 5, exit 3 minutes. Advanced route: inquiry 18, organise 4, selected practice 13, exit 5 minutes.

Prerequisites: Solve a linear inequality, mark open/closed endpoints and distinguish real values from whole-item counts.

Ordinary route: use the first fixed case for the five-minute demonstration. Advanced route: complete the full investigation.

Inquiry and discussion: 18 minutes using tariffs: A=2n, B=10+n, minimum 8 visits, budget 25. Predict which plan is suitable at n=9 and n=14 (3 min). Calculate and record both price and feasibility, then solve each plan’ s budget and minimum restrictions on paper (7 min). Hold all prices/minimum fixed and change only budget to 15; check whether either overlap remains (5 min). Explain why “cheaper” and “feasible” are different, with exact integer endpoints (3 min). The existing controls support these parameter values.

Worked example 1

A club pays a fixed 5 dollars plus 2 dollars per visit, has at most 27 dollars and needs at least 3 visits. Let x be the integer visit count. Form the restrictions and find all feasible counts.

Reference answer

$3 \le x \le 11, x \in \mathbb{Z}$.

$$3 \le x \le 11$$

1. Cost gives $2x + 5 \le 27$, and need gives $x \ge 3$.
2. The cost limit is $x \le 11$; intersect to get $3 \le x \le 11$.
3. Every integer from 3 through 11 is feasible; both extreme counts meet the budget.

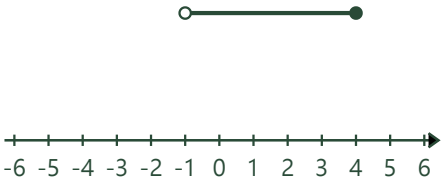
- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Guided attempt 2

Solve the system of inequalities; require every condition to hold.

$$\begin{cases} x > -1 \\ x \le 4 \end{cases}$$

1. The first condition keeps values to the ____ of −1; that endpoint is open.
2. The second keeps values at or to the ____ of 4; that endpoint is filled.
3. Complete the overlap $-1 \square x \square 4$ and draw it.



Use the stated conditions and the labelled scales.

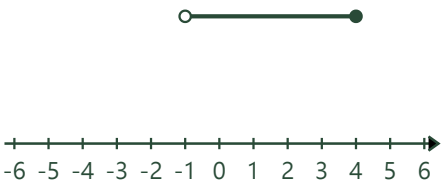
Reference answer

$-1 < x \le 4$.

$$-1 < x \le 4$$

1. Solve each original inequality separately, reversing direction only when dividing by a negative number.
2. Take their common part: $-1 < x \le 4$.
3. Check every included boundary in the original inequalities.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.



Use the stated conditions and the labelled scales.

Prerequisite 3

"x is at most 7" means which inequality?

A. $x \ge 7$

B. $x < 7$

C. $x \le 7$

Reference answer

$x \le 7$

1. At most allows 7 but excludes larger values.
2. Thus x≤7.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Prerequisite 4

Is 2 included in $2 < x \leq 5$?

A. Yes; both endpoints are always included.

B. No; the lower inequality is strict.

Reference answer

No; the lower inequality is strict.

1. Substituting 2 gives 2<2, which is false.
2. Use an open lower endpoint.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 5

Solve the system of inequalities; require every condition to hold.

$$\begin{cases} 3x - 2 > 4 \\ 2x + 1 \leq 11 \end{cases}$$



Use the stated conditions and the labelled scales.

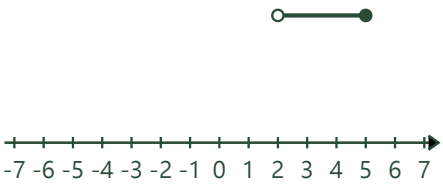
Reference answer

$2 < x \leq 5$.

$$2 < x \leq 5$$

1. The first gives $x > 2$; the second gives $x \leq 5$.
2. Both hold exactly on $2 < x \leq 5$.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.



Use the stated conditions and the labelled scales.

Independent practice 6

Solve the system of inequalities; require every condition to hold.

$$\begin{cases} x > 4 \\ x \le 2 \end{cases}$$

Reference answer

$\varnothing$.

$$\varnothing$$

1. One condition requires x greater than 4, the other at most 2.
2. No real number meets both; the solution set is empty.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 7

Notebooks cost 3 dollars each with one fixed 2-dollar wrapping fee. With 20 dollars, buy at least one notebook. Let x be the count; find all feasible counts.

Reference answer

$1 \le x \le 6, x \in \mathbb{Z}$.

$$1 \le x \le 6$$

1. The restrictions are $3x + 2 \le 20$ and $x \ge 1$.
2. Cost gives x≤6; integer counts are 1,2,3,4,5,6.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 8

Solve the system of inequalities; require every condition to hold.

$$\begin{cases} x \ge 3 \\ x \le 3 \end{cases}$$

Reference answer

$x = 3$.

$$x = 3$$

1. The number is at least 3 and at most 3.
2. Only x=3 meets both; the endpoint is included in each condition.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Exit question 9

List the integer solutions of $\frac{5}{2} < x < 6$.

$$\begin{cases} x > 5/2 \\ x < 6 \end{cases}$$

Reference answer

$3 \leq x \leq 5, x \in \mathbb{Z}$.

$$\frac{5}{2} < x < 6$$

1. Solve each original inequality separately, reversing direction only when dividing by a negative number.
2. Take their common part: $\frac{5}{2} < x < 6$.
3. Apply the stated integer restriction and check the extreme allowed integers.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Exit question 10

A costs 2n dollars; B costs 10+n dollars. For n=14 and budget 25 dollars, which is feasible?

A. Only B: 24 dollars; A costs 28.
B. Both, because B is cheaper.
C. Only A.

Reference answer

Only B: 24 dollars; A costs 28.

1. A costs 28, exceeding 25.
2. B costs 24, within the budget.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 11

Solve the system of inequalities; require every condition to hold.

$$\begin{cases} x \geq 1 \\ x > -2 \end{cases}$$

Reference answer

$x \geq 1$.

$$x \geq 1$$

1. Solve each original inequality separately, reversing direction only when dividing by a negative number.
2. Take their common part: $x \geq 1$.
3. Check every included boundary in the original inequalities.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 12

An item costs 80 dollars. Let x be its selling price. The profit must be at least 15% of cost. Form an inequality and find the minimum price.

Reference answer

$x \geq 92$.

$$x \geq 92$$

1. Profit divided by cost gives $\frac{x-80}{80} \geq \frac{15}{100}$.
2. Multiply by positive 80: $x - 80 \geq 12$, so $x \geq 92$.
3. The minimum selling price is 92 dollars.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 13

A container has capacity 11 L, initially 3 L. Each full bottle adds 2 L. Let x be a nonnegative integer bottle count. Find all counts that fit.

Reference answer

$0 \leq x \leq 4, x \in \mathbb{Z}$.

$$0 \leq x \leq 4$$

1. Use $2x + 3 \leq 11$ and $x \geq 0$.
2. Thus 0≤x≤4; counts 0 through 4 fit.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 14

Solve the system of inequalities; require every condition to hold.

$$\begin{cases} x < 3 \\ x \geq 3 \end{cases}$$

Reference answer

$\varnothing$.

$$\varnothing$$

1. Solve each original inequality separately, reversing direction only when dividing by a negative number.
2. Take their common part: $\varnothing$.
3. Check every included boundary in the original inequalities.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 15

Find the integer solutions of $2 < x < 4$. Explain why its real solution set is not a single point.

$$\begin{cases} x > 2 \\ x < 4 \end{cases}$$

Reference answer

$x = 3, x \in \mathbb{Z}$.

$$2 < x < 4$$

1. The only integer strictly between 2 and 4 is 3.
2. Real numbers such as 2.5 and 3.5 also satisfy the inequalities, so the real interval contains more than one point.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 16

There are 29 toys and x children. Giving 3 each leaves at least 8 toys; giving 5 each would be short by 1 to 4 toys inclusive. Find the positive integer child count.

Reference answer

$x = 6, x \in \mathbb{Z}$.

$$6 \leq x \leq \frac{33}{5}$$

1. The three restrictions are $29 - 3x \geq 8, 5x - 29 \geq 1, 5x - 29 \leq 4$.
2. They give $x \leq 7$ and $6 \leq x \leq \frac{33}{5}$.
3. The only positive integer is 6; it leaves 11 with three each and is short 1 with five each.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.