Grouped data and histograms: frequency and area — Teacher answers

Year 7, lower volume • text pp. 95–98 • textbook lessons 48–49

Use the worked and guided tasks selectively. The eight core paper tasks are prerequisites, independent tasks and exits; further practice and extensions are optional.

Ordinary route: prerequisites 4, short inquiry 5, worked/guided 8, independent 15, correction 5, exit 3 minutes. Advanced route: inquiry 18, organise 4, selected practice 13, exit 5 minutes.

Prerequisites: Read frequencies, compare interval boundaries and calculate widths, ratios and rectangle areas.

Ordinary route: use the first fixed case for the five-minute demonstration. Advanced route: complete the full investigation.

Inquiry and discussion: 18 minutes, fixed data on paper: use 1,1,1,1,2,2,3,3,3,3,4,5. Predict whether regrouping changes the sample size (3 min). Tally widths 2 over [0,2),[2,4),[4,6], then widths 3 over [0,3),[3,6]; compare frequencies 4,6,2 versus 6,6 and density heights (7 min). For a second given table, compare widths 10 and 20 with frequencies 20 and 30: calculate heights 2 and 3/2 and recover counts through areas (5 min). Conclude which totals stay fixed and why a shorter density bar can contain more observations (3 min).

Worked example 1

Group the data $1, 1, 1, 1, 2, 2, 3, 3, 3, 3, 4, 5$ into $0 \le x < 2, 2 \le x < 4, 4 \le x \le 6$. Give frequencies and density heights (frequency ÷ class width).

Interval	Frequency	Density
[0,2)	_____	_____
[2,4)	_____	_____
[4,6]	_____	_____

Use the stated conditions and the labelled scales.

Reference answer

Frequencies 4,6,2; width 2 each; heights 2,3,1.

$$(h_1, h_2, h_3) = (2, 3, 1)$$

1. Place each boundary in the class indicated by its inequality.
2. Counts are 4,6,2 and sum to 12.
3. Divide each frequency by width 2; bar area then equals frequency.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

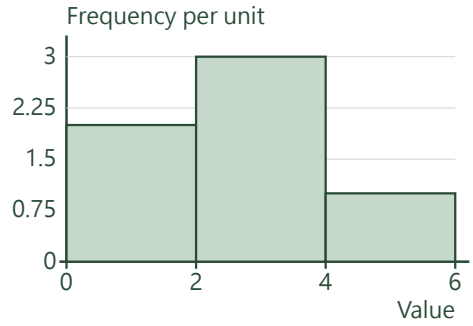
Interval	Frequency	Density
[0,2)	4	2
[2,4)	6	3
[4,6]	2	1

Use the stated conditions and the labelled scales.

Guided attempt 2

Draw a density histogram for $0 \le x < 2$: 4; $2 \le x < 4$: 6; $4 \le x \le 6$: 2. Label frequency per unit on the vertical axis.

1. All class widths equal $2 - 0 = \square$.
2. Complete heights $h_1 = \frac{4}{2} = \square,\ h_2 = \frac{6}{2} = \square,\ h_3 = \frac{2}{2} = \square$.
3. Draw touching bars over the three intervals; label y as frequency per unit and check area total _____.



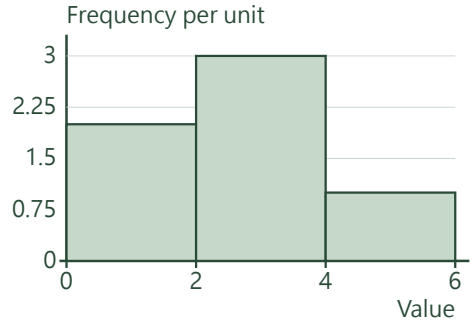
Use the stated conditions and the labelled scales.

Reference answer

Adjacent bars on [0,2),[2,4),[4,6] have heights 2,3,1.

1. The class widths are all 2.
2. Heights are 4/2=2, 6/2=3, 2/2=1.
3. Use touching bars and retain the class boundaries.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.



Use the stated conditions and the labelled scales.

Prerequisite 3

For data 4,7,9,12, find the maximum minus minimum.

Reference answer

8.

$$12 - 4$$

1. The maximum is 12 and minimum 4.
2. The difference is 8.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Prerequisite 4

A class contains 3 of 12 observations. Find its relative frequency.

Reference answer

$\frac{1}{4}$.

$$(3/12)\cdot 1$$

1. The given categories account for every observation exactly once.
2. Use $\frac{3}{12}$ as the relative frequency.
3. The requested result is $\frac{1}{4}$.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 5

Data are $0, 1, 2, 2, 3, 4, 5, 6$. Use classes $0 \le x < 2, 2 \le x < 4, 4 \le x \le 6$. Give all three frequencies.

Reference answer

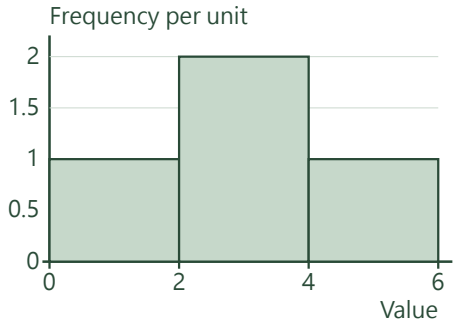
2,3,3.

1. The two 2s go in the second class; 4 goes in the third.
2. The final class includes 6. Counts 2+3+3=8 retain every observation exactly once.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 6

The given density histogram has class width 2 throughout and heights 1,2,1. What fraction of the observations have value at least 2?



Use the stated conditions and the labelled scales.

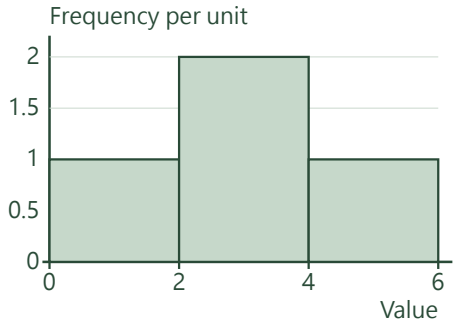
Reference answer

$\frac{3}{4}$

$$\frac{3}{4}$$

1. Multiply heights by widths to recover frequencies 2,4,2.
2. The final two classes contain 6 of the total 8, so 6/8=3/4.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.



Use the stated conditions and the labelled scales.

Independent practice 7

Class A spans 0 to 10 with frequency 20. Class B spans 10 to 30 with frequency 30. Find density heights for a histogram. Which class has more observations?

Reference answer

Heights 2 and 3/2; B has more observations although its bar is shorter.

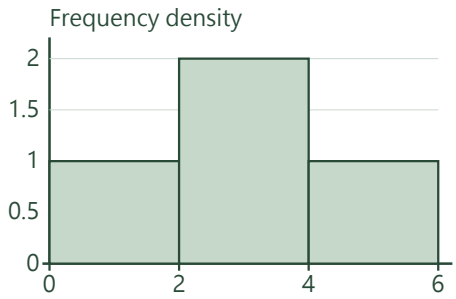
$$h_A = 2, \quad h_B = \frac{3}{2}$$

1. Widths are 10 and 20.
2. Heights are 20/10=2 and 30/20=3/2; compare frequencies 20 and 30 for counts.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Independent practice 8

Draw a density histogram for $0 \le x < 2$: 2, $2 \le x < 4$: 4, $4 \le x \le 6$: 2. Label both axes.



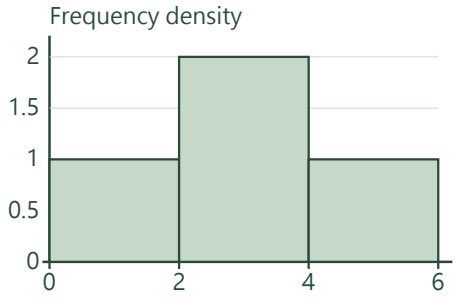
Use the stated conditions and the labelled scales.

Reference answer

Three touching bars have heights 1,2,1; horizontal axis is value, vertical axis frequency per unit.

1. All widths are 2, so divide frequencies by 2.
2. Draw heights 1,2,1 on their stated intervals; their areas sum to 8.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.



Use the stated conditions and the labelled scales.

Exit question 9

For classes $0 \le x < 2$ and $2 \le x < 4$, where is x=2 counted?

A. Both classes.
B. Neither class.
C. Only the second class.

Reference answer

Only the second class.

1. The first class excludes its upper boundary 2.
2. The second includes its lower boundary, so count 2 once there.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Exit question 10

A density histogram bar has width 4 and height 3 observations per unit. Find its frequency.

Reference answer

12.

$$4 \cdot 3$$

1. Frequency equals width times density height.
2. 4×3=12 observations.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 11

Three nonoverlapping classes have frequencies 4,7,5. Find the total sample size.

Reference answer

16.

$$4+7+5$$

1. The given categories account for every observation exactly once.
2. Add the mutually exclusive category counts: $4+7+5$.
3. The requested result is 16.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 12

6 of 20 observations fall in one class. Find its percentage.

Reference answer

30 %.

$$(6/20)\cdot 100$$

1. The given categories account for every observation exactly once.
2. Use $\frac{6}{20}\times 100\%$ to express a percentage.
3. The requested result is 30.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 13

Classes [0,2), [2,4), [4,6] have frequencies 2,4,2. For the frequency-distribution line graph, give the three class-midpoint/frequency points to connect in order.

Reference answer

$(1,2),(3,4),(5,2)$

1. Class midpoints are (0+2)/2=1, (2+4)/2=3, (4+6)/2=5.
2. Pair them with frequencies 2,4,2 and connect in class order.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Further practice 14

Choose the classes [0,5), [5,10), [10,15), [15,20] for values between 1 and 18. How many classes are there, and what is the common class width?

Reference answer

Four classes, each width 5.

1. Read successive boundaries 0,5,10,15,20.
2. There are four intervals and every adjacent boundary difference is 5.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 15

One density bar spans 0–10 with height 1. A second spans 10–40 with height 1/2. Which contains more observations? Explain why comparing height alone fails.

Reference answer

The second: areas/frequencies are 10 and 15.

1. Multiply each height by its own width: $1\times10=10$ and $(1/2)\times30=15$.
2. Density heights alone are not comparable as frequencies when widths differ.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.

Extension 16

Use the same data $1, 1, 1, 1, 2, 2, 3, 3, 3, 3, 4, 5$. Regroup into $0 \leq x < 3$ and $3 \leq x \leq 6$. Find frequencies and explain what total is preserved.

Reference answer

6 and 6; total observations remain 12, while the distribution’ s appearance changes.

1. The first class contains four 1s and two 2s.
2. The remaining six values form the second class; 6+6=12.

- Use the given conditions and show a justified intermediate step.
- State and check the requested result accurately.