Equations: meaning, solutions and equal operations — Teacher answers

Year 7, upper volume • text pp. 55–58 • foundation lessons 28–29

Use the prerequisite checks to choose support, project the worked and guided items, then let learners work on paper. Select supplements or extensions as needed. No learner answers are saved by this presentation.

Worked example 1

Is x = 4 a solution of the equation? Calculate both original sides to justify your answer.

$$3x + 2 = 14$$

Reference answer

Yes. The left side is 3 × 4 + 2 = 14 and the right side is 14.

$$x = 4$$

1. Substitute the proposed value into the entire left side.
2. Both sides equal 14, so the equality holds.

- Use the original equality and apply each operation to both sides.
- Obtain the solution and check it in the original equation or situation.

Guided attempt 2

Three identical notebooks and a single 2-dollar packing fee cost 17 dollars. Let x be the price of one notebook. Write an equation and find x.

1. Notebook cost = ____; total equation: ____ = 17.
2. After subtracting 2: ____ x = ____.
3. x = ____ dollars; check: ____.

Reference answer

3x + 2 = 17; x = 5 dollars.

$$x = 5$$

1. The repeated cost is 3x; add the fixed fee once: 3x + 2 = 17.
2. Subtract 2 from both sides: 3x = 15. Divide by 3: x = 5.
3. Check: 3 × 5 + 2 = 17.

- Use the original equality and apply each operation to both sides.
- Obtain the solution and check it in the original equation or situation.

Prerequisite 3

Is 2x + 3, by itself, an equation in an unknown? Identify the missing feature.

Reference answer

No. It is an expression; there is no equality sign joining two sides.

1. The letter x is present.
2. An equation also requires an equality, which is absent here.

- State the applicable definition or condition accurately.
- Support the conclusion with the given numbers, a check or a concrete counterexample.

Prerequisite 4

Is 7 + 5 = 12 an equation containing an unknown, as required by this lesson’ s definition? Explain.

Reference answer

It is a true equality, but it contains no unknown, so it is not an equation in an unknown in this lesson.

1. The equality sign is present and the two sides agree.
2. No unknown quantity appears.

- State the applicable definition or condition accurately.
- Support the conclusion with the given numbers, a check or a concrete counterexample.

Independent practice 5

Solve, then verify by substitution.

$$2x - 3 = 7$$

Reference answer

x = 5.

$$x = 5$$

1. Add 3 on both sides: 2x = 10.
2. Divide by 2: x = 5; check 2 × 5 − 3 = 7.

- Use the original equality and apply each operation to both sides.
- Obtain the solution and check it in the original equation or situation.

Independent practice 6

From x + 6 = 10, a learner subtracts 6 from both sides and writes x = 4. Does this preserve the solutions? Explain.

Reference answer

Yes. Both sides have the same subtraction, and both equations have solution x = 4.

1. (x + 6) − 6 = 10 − 6.
2. Simplifying gives x = 4.

- State the applicable definition or condition accurately.
- Support the conclusion with the given numbers, a check or a concrete counterexample.

Independent practice 7

From x/3 = 4, a learner multiplies both sides by 3 and obtains x = 12. Is this equivalent?

Reference answer

Yes. Multiplication by the same nonzero number 3 preserves the solution x = 12.

1. Multiply the complete left and right sides by 3.
2. The left becomes x and the right becomes 12.

- State the applicable definition or condition accurately.
- Support the conclusion with the given numbers, a check or a concrete counterexample.

Independent practice 8

From x + 3 = 8, a learner subtracts 3 only on the left and writes x = 8. Explain why this does not preserve the equation.

Reference answer

The right side must also lose 3. The correct result is x = 5. At x = 8, the original left side is 11, not 8.

1. The valid operation is (x + 3) − 3 = 8 − 3.
2. It gives x = 5. The proposed x = 8 fails the original equation.

- State the applicable definition or condition accurately.
- Support the conclusion with the given numbers, a check or a concrete counterexample.

Exit question 9

A number plus 7 equals 19. Let the number be x, write an equation and find it.

Reference answer

x + 7 = 19, so x = 12.

$$x = 12$$

1. Translate the sentence directly: x + 7 = 19.
2. Subtract 7 on both sides: x = 12. Check 12 + 7 = 19.

- Use the original equality and apply each operation to both sides.
- Obtain the solution and check it in the original equation or situation.

Exit question 10

Verify whether x = −2 solves the equation. Show the original left and right sides.

$$4x + 1 = -7$$

Reference answer

Yes: 4 × (−2) + 1 = −7, equal to the right side.

$$x = -2$$

1. Substitute −2 with its sign: 4 × (−2) + 1.
2. The left side is −8 + 1 = −7.

- Use the original equality and apply each operation to both sides.
- Obtain the solution and check it in the original equation or situation.

Further practice 11

Does 5x + 2 = 17 fit the definition of an equation in an unknown? Identify both required features.

Reference answer

Yes. It has an equality sign and an unknown x.

1. The equality sign joins two expressions.
2. The left side contains unknown x.

- State the applicable definition or condition accurately.
- Support the conclusion with the given numbers, a check or a concrete counterexample.

Further practice 12

From x − 4 = 9, add 4 to both sides. Is the result x = 13 equivalent?

Reference answer

Yes. The same addition removes −4 from the left and gives 13 on the right.

1. (x − 4) + 4 = 9 + 4.
2. Both equations have solution x = 13.

- State the applicable definition or condition accurately.
- Support the conclusion with the given numbers, a check or a concrete counterexample.

Further practice 13

From −2x = 10, divide both sides by −2. Explain why x = −5 is equivalent.

Reference answer

−2 is nonzero. Dividing both complete sides by −2 gives x = −5 with the same solution.

1. The left coefficient becomes 1.
2. 10 ÷ (−2) = −5; substitution gives 10.

- State the applicable definition or condition accurately.
- Support the conclusion with the given numbers, a check or a concrete counterexample.

Further practice 14

Multiplying x = 3 by 0 gives 0 = 0. Do these two equations have the same solutions? Give a concrete test value.

Reference answer

No. For example, x = 4 satisfies 0 = 0 but not x = 3. Multiplying by zero loses the original restriction.

1. The original equation allows only x = 3.
2. The new identity is true for every x, including 4.

- State the applicable definition or condition accurately.
- Support the conclusion with the given numbers, a check or a concrete counterexample.

Extension 15

Does every real value of x solve x + x = 2x? Explain without choosing just one test value.

$$x + x = 2x$$

Reference answer

Yes. Combining the identical terms gives 2x = 2x for every real x.

1. The left side is 2x by the definition of adding like terms.
2. Both sides are the same expression for every real value.

- Explain equality of the two expressions for every real value, not just a numerical trial.

Extension 16

Can 2x + 1 = 2x + 4 have a solution? Apply the same subtraction on both sides and explain.

$$2x + 1 = 2x + 4$$

Reference answer

No solution: subtracting 2x on both sides would require 1 = 4.

1. Subtract 2x on both sides.
2. The false statement 1 = 4 cannot be repaired by choosing x.

- Derive the contradiction and conclude no solution.