Equations in context: quantities, rates and conditions — Teacher answers

Year 7, upper volume • text pp. 67–78 • foundation lessons 34–39

Use the prerequisite checks to choose support, project the worked and guided items, then let learners work on paper. Select supplements or extensions as needed. No learner answers are saved by this presentation.

Worked example 1

Two groups collected 32 cards in total. The larger group collected 8 more than the smaller. Let x be the smaller group’s number. Find both numbers.

Reference answer

x = 12 cards; the larger group has 20 cards.

$$x = 12$$

1. The larger number is x + 8, so x + (x + 8) = 32.
2. 2x + 8 = 32 gives x = 12; the other number is 20.
3. Check: 12 + 20 = 32 and 20 − 12 = 8.

- Define x with its unit and write an equality matching the situation.
- Solve, then check the original quantities, units and any range or whole-number condition.

Guided attempt 2

A walker moves at 60 m/min. A runner starts from the same point 2 minutes later at 80 m/min in the same direction. Speeds stay constant. Let x be the runner’s time until catching up. Find x.

1. Walker: time ____ min; distance ____ m.
2. Runner: time ____ min; distance ____ m.
3. Equal-distance equation: ____.

Reference answer

x = 6 minutes. The walker travels for 8 minutes; both cover 480 m.

$$x = 6$$

1. The walker’s time is x + 2, whereas the runner’s time is x.
2. Equal distances give 60(x + 2) = 80x.
3. 60x + 120 = 80x gives x = 6; check 60 × 8 = 80 × 6 = 480.

- Define x with its unit and write an equality matching the situation.
- Solve, then check the original quantities, units and any range or whole-number condition.

Prerequisite 3

One worker completes 1/6 of a job each hour and another completes 1/3. Find their combined hourly rate.

$$\frac{1}{6} + \frac{1}{3}$$

Reference answer

1/2 of the job per hour.

$$\frac{1}{2}$$

1. The rates use the same whole job and the same hour unit.
2. 1/6 + 1/3 = 1/6 + 2/6 = 1/2.

- Apply the relevant sign and order rules to every operation.
- Show an intermediate calculation and obtain the exact stated value.

Prerequisite 4

Profit is 20% of a 50-dollar cost. Find the profit, not the selling price.

$$\frac{20}{100} \times 50$$

Reference answer

10 dollars of profit; the cost is the percentage base.

$$10$$

1. 20% means 20/100 of the cost.
2. Multiply by 50 to get 10.

- Apply the relevant sign and order rules to every operation.
- Show an intermediate calculation and obtain the exact stated value.

Independent practice 5

Notebooks cost 4 dollars each, plus a single 3-dollar packing fee. The total is 27 dollars. Let x be the number of notebooks. Find x.

Reference answer

6 notebooks.

$$x = 6$$

1. 4x + 3 = 27.
2. 4x = 24 gives x = 6. Check 4 × 6 + 3 = 27.

- Define x with its unit and write an equality matching the situation.
- Solve, then check the original quantities, units and any range or whole-number condition.

Independent practice 6

Two workers would finish the same job alone in 4 hours and 6 hours. They work together at constant independent rates. Let x be their time together. Find x.

Reference answer

12/5 hours, or 2.4 hours.

$$x = \frac{12}{5}$$

1. Each contribution is rate × time: x/4 and x/6; their sum is one job.
2. Multiply x/4 + x/6 = 1 by 12: 3x + 2x = 12, so x = 12/5.
3. Check: 3/5 + 2/5 = 1, and the time is shorter than either solo time.

- Define x with its unit and write an equality matching the situation.
- Solve, then check the original quantities, units and any range or whole-number condition.

Independent practice 7

A product sells for 60 dollars at a profit of 20% of its cost. Let the cost be x dollars. Find x.

Reference answer

The cost is 50 dollars.

$$x = 50$$

1. Selling price = cost + 20% of cost = 1.2x.
2. 1.2x = 60 gives x = 50. Check 50 + 10 = 60.

- Define x with its unit and write an equality matching the situation.
- Solve, then check the original quantities, units and any range or whole-number condition.

Independent practice 8

The ratio of boys to girls is 2:3 and there are 35 pupils. Let x be the number of boys. Find x and check both counts.

Reference answer

14 boys and 21 girls.

$$x = 14$$

1. Girls number (3/2)x, so x + (3/2)x = 35.
2. (5/2)x = 35 gives x = 14; girls = 21.
3. Check 14 + 21 = 35 and 14:21 = 2:3.

- Define x with its unit and write an equality matching the situation.
- Solve, then check the original quantities, units and any range or whole-number condition.

Exit question 9

An item marked at 80 dollars is sold at 25% off. That selling price is 20% above cost. Let cost be x dollars. Find x and identify both percentage bases.

Reference answer

Cost = 50 dollars. Discount uses the marked price; profit uses the cost.

$$x = 50$$

1. Selling price = 80 × 75% = 60 dollars.
2. 60 = 1.2x, so x = 50.
3. Check 50 × 120% = 60 and 80 × 75% = 60.

- Define x with its unit and write an equality matching the situation.
- Solve, then check the original quantities, units and any range or whole-number condition.

Exit question 10

The first 20 units cost 2 dollars each; additional units cost 3 dollars each. A bill is 49 dollars. Let x be total units used. Choose the correct branch, solve and check its condition.

Reference answer

23 units. The first 20 cost 40 dollars and the extra 3 cost 9 dollars.

$$x = 23$$

1. 49 exceeds the first-block bill of 40, so use x > 20.
2. 40 + 3(x − 20) = 49 gives x − 20 = 3 and x = 23.
3. Check the bill and the condition 23 > 20.

- Define x with its unit and write an equality matching the situation.
- Solve, then check the original quantities, units and any range or whole-number condition.

Further practice 11

Two collections total 26 items. One has 6 more than the other. Let x be the smaller count. Find both.

Reference answer

10 items and 16 items.

$$x = 10$$

1. x + (x + 6) = 26 gives 2x = 20.
2. x = 10; check 10 + 16 = 26 and 16 − 10 = 6.

- Define x with its unit and write an equality matching the situation.
- Solve, then check the original quantities, units and any range or whole-number condition.

Further practice 12

A rectangle is 3 cm longer than it is wide and has perimeter 30 cm. Let x be its width. Find the dimensions.

Reference answer

Width 6 cm and length 9 cm.

$$x = 6$$

1. There are two widths and two lengths: 2x + 2(x + 3) = 30.
2. 4x + 6 = 30 gives x = 6.
3. Check 2 × 6 + 2 × 9 = 30.

- Define x with its unit and write an equality matching the situation.
- Solve, then check the original quantities, units and any range or whole-number condition.

Further practice 13

A slower runner is 60 m ahead. Speeds remain 3 m/s and 5 m/s in the same direction. Let x be the faster runner’s catching time in seconds. Find x.

Reference answer

30 seconds.

$$x = 30$$

1. The gap closes at 5 − 3 = 2 m/s.
2. 2x = 60 gives x = 30.
3. Check the faster distance exceeds the slower distance by 60 m.

- Define x with its unit and write an equality matching the situation.
- Solve, then check the original quantities, units and any range or whole-number condition.

Further practice 14

Two workers take 3 hours and 6 hours alone for the same job. At constant additive rates, how long do they take together? Let the time be x hours.

Reference answer

2 hours.

$$x = 2$$

1. x/3 + x/6 = 1.
2. Multiply by 6: 2x + x = 6, so x = 2.
3. Check 2/3 + 2/6 = 1.

- Define x with its unit and write an equality matching the situation.
- Solve, then check the original quantities, units and any range or whole-number condition.

Extension 15

Eight tickets cost 200 dollars. Adult tickets cost 30 dollars and student tickets 20 dollars. Let x be the adult-ticket count. Find both counts and check that they are possible.

Reference answer

4 adult tickets and 4 student tickets.

$$x = 4$$

1. The student-ticket count is 8 − x.
2. 30x + 20(8 − x) = 200 gives 10x = 40 and x = 4.
3. Both counts are whole numbers between 0 and 8; cost is 120 + 80 = 200.

- Define x with its unit and write an equality matching the situation.
- Solve, then check the original quantities, units and any range or whole-number condition.

Extension 16

Use the same tariff: first 20 units at 2 dollars, extra units at 3 dollars. The bill is only 30 dollars. Find usage x and explain why the extra-unit formula would use the wrong branch.

Reference answer

15 units, within $0 \leq x \leq 20$. The extra-unit formula assumes $x > 20$, which is not satisfied.

$$x = 15$$

1. The first-block maximum bill is $20 \times 2 = 40$ dollars; 30 is below it.
2. Use $2x = 30$, giving $x = 15$.
3. Check $15 \leq 20$ and $2 \times 15 = 30$.

- Define x with its unit and write an equality matching the situation.
- Solve, then check the original quantities, units and any range or whole-number condition.