Motivation for Simulation

When is simulation indispensable?

- Unsafe, unethical or costly physical experiments
- Intractable mathematics for non-linear/high-dimensional systems
- Rapid "what-if" analysis before major investments

Motivation for Simulation

Core advantages

- Full control of virtual laboratory inputs
- Quantified uncertainty via repeated replications
- Insight into emergent or rare events

Motivation for Simulation

Key Idea

Simulation extends mathematics from *deduction* to *experimentation*.

Stochastic Differential Equation (SDE) Basics

$$dX_t = \mu(X_t, t)\,dt + \sigma(X_t, t)\,dW_t$$

- μ – deterministic drift
- σ – diffusion amplitude

Stochastic Differential Equation (SDE) Basics

Euler–Maruyama discretisation

$$X_{k+1} = X_k + \mu_k h + \sigma_k \sqrt{h}\, \xi_k, \qquad \xi_k \sim N(0,1)$$

- Strong convergence order $1/2$
- Higher-order schemes (Milstein, etc.) give better accuracy at extra cost

Plain Monte Carlo

$$\hat{I}_n = \frac{|D|}{n} \sum_{i=1}^{n} f(X_i), \qquad X_i \sim \mathrm{Unif}(D)$$

- Unbiased; Var $\propto 1/n$
- Convergence independent of dimension but relatively slow

Plain Monte Carlo

Variance-reduction toolbox

- Antithetic sampling
- Control variates
- Importance sampling
- Stratified / Latin-hypercube sampling

DES – Key Notions

Definition (Discrete-Event System)

$\big(\mathcal{S},\ \mathcal{E},\ \delta,\ \text{scheduler}\big)$ with state jumps only at event times.

DES – Key Notions

Typical workflow

1. Initialise state and future-event list
2. Pop next event, advance clock
3. Execute δ, schedule new events
4. Collect statistics, repeat

DES – Key Notions

Representative domains

- Queueing in call centres
- Hospital patient flow
- Supply-chain logistics

Validation & Verification Flow

Verification – are we solving the equations right?

- Unit tests, time-step sensitivity, conservation checks

Validation & Verification Flow

Validation – are we solving the right equations?

- Goodness-of-fit tests (KS, χ^2)
- Expert face-validity reviews
- Historical data comparison

Validation & Verification Flow

Uncertainty analysis

- Replication confidence intervals
- Bootstrap or jack-knife error bars
- Global sensitivity indices (Sobol)

Five-Step Workflow Recap

1. **Problem formulation**

Five-Step Workflow Recap

2. Model construction

Five-Step Workflow Recap

3. Algorithm design

Five-Step Workflow Recap

4. Computational experiment

Five-Step Workflow Recap

5. Analysis & V&V

Key Takeaways

- Simulation is a bridge between mathematical theory and real-world experimentation.
- Deterministic, stochastic, agent-based and event-driven paradigms cover most systems.
- Monte Carlo is the universal numerical engine; variance-reduction boosts efficiency.
- Rigorous V&V underpins model credibility.

Questions?